GRADED WORK IN ARITHMETIC

EIGHTH YEAR

BAIRD

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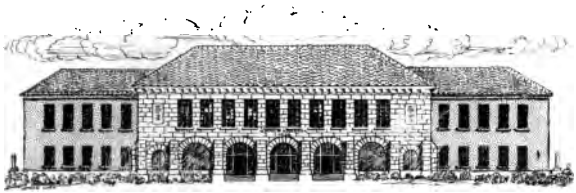

GRADED WORK

IN

ARITHMETIC

BY

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EIGHTH YEAR

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GRADED WORK IN ARITH. VIII.
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NOTE

THIS book is designed to complete a well-graded and comprehensive grammar school course in arithmetic.

It begins with a review of the essential parts of the lower books, and then furnishes a thorough treatment of further applications of percentage and interest required in the grammar school course. It includes also exercises on the metric system, the elementary principles of algebra, involution, evolution, and mensuration, besides a number of special problems on work, time, temperature, specific gravity, etc.

Like the other books of the series, this book is largely based on the inductive method of teaching.

Great care has been taken in the selection of illustrative examples. Operations, explanations, and analyses have been given in full, in order that the principles involved may be easily and clearly understood by the pupil with but little assistance from the teacher.

CONTENTS

REVIEW WORK

LESSON 1

1. How many yards of tape can be bought for \$5, at $6\frac{1}{4}¢$ a yard? At $8\frac{1}{3}¢$ a yard? At $33\frac{1}{3}¢$ a yard?

2. If $\frac{3}{4}$ of a ton of coal costs $\$4\frac{1}{5}$, how much is one ton worth?

3. At $\$\frac{3}{8}$ a yard, what part of a yard of lace can be bought for $\$\frac{3}{16}$?

4. The sum of two fractions is $\frac{26}{27}$, and one of them is $\frac{2}{9}$. What is the other fraction?

5. How many half-inch cubes are there in a board 1 ft. square and 1 in. thick?

6. If $\frac{3}{4}\%$ of my money is \$9.60, what is my money?

7. 70% of a class of 40 pupils were promoted. How many were not promoted?

8. State quickly the fractional equivalents of the following: 20%, 25%, 40%, 50%, 80%, $33\frac{1}{3}\%$, $66\frac{2}{3}\%$, $12\frac{1}{2}\%$, $37\frac{1}{2}\%$, $16\frac{2}{3}\%$, $8\frac{1}{3}\%$, $87\frac{1}{2}\%$, $62\frac{1}{2}\%$, $83\frac{1}{3}\%$.

9. If $\frac{5}{14}$ of a pole stands in the water, $\frac{1}{7}$ in the mud, and $12\frac{1}{2}$ ft. in the air, how long is the pole?

10. What is a circle? What is the rule for finding the circumference when the diameter is given? How do you find the diameter when the circumference is given?

11. If a man can walk 10 miles in $3\frac{1}{3}$ hours, how far can he walk in 5 hours?

12. What is $\frac{5}{8}$ of 2? 70% of 7? $37\frac{1}{2}\%$ of 3? $87\frac{1}{2}\%$ of 5?

13. A boy spent $\frac{1}{2}$ of his money for a knife, and $\frac{1}{3}$ of the remainder for a top. What per cent of his money had he left?

14. A can build $\frac{1}{4}$ of a wall in 8 days. What part of it can he build in 1 day? If B can build $\frac{1}{3}$ of the same wall in 10 days, what part of the wall can he build in 1 day? What part of the wall can A and B, working together, build in 1 day?

LESSON 2

1. If .75 of a ton of clover hay is worth $\$11.62\frac{1}{2}$, find the value of 8640 lb.

2. A boy lost $\frac{3}{5}$ of his money, and afterwards earned $\frac{1}{3}$ of the amount he had lost. What part of the original amount had he then? What per cent?

3. If $.8\frac{3}{4}$ of a yard of flannel is worth \$1.19, how many yards are worth \$13.60?

4. I sold my sleigh to Mr. Randall for $1\frac{1}{4}$ times what it cost me; Mr. Randall sold it to Mr. Detrick for \$32, which was $\frac{1}{9}$ less than he paid for it. How much did the sleigh cost me?

5. A owns $32\frac{3}{5}$ acres of land, B owns $45\frac{1}{2}$ acres, and C owns half as many acres as A and B together. How many acres does C own, and how many acres have they all?

6. Find the perimeter of a sector of 45° of a circle if the radius is 20 inches. Find the area of the sector.

7. Find the value of a piece of land 36 chains long and 24 chains wide at $40 an acre.

8. A druggist bought a pound of quinine, and sold from it 8 ℥ 4 ʒ 2 ℈ 16 gr. How many 2-grain pills will the remainder make?

9. A man gave his son 128 A. 80 P. of land, which was $66\frac{2}{3}\%$ of what he had left. How many acres had he before he made this gift to his son?

10. Find the interest of $380 from Jan. 1, 1901, to Dec. 16, 1901, at 6%.

11. Charles Weaver owes James Birmingham $460, payable in 60 da., at 5%. Write a promissory note, supplying *place* and *date of writing*. Find the amount of the note at maturity.

12. At $1\frac{1}{2}\%$ per annum, how much must be paid in 3 yr. for the insurance of $\frac{2}{3}$ of the value of a house worth $6000?

13. How many square feet of galvanized iron are sufficient to make a smokepipe for a furnace, if the pipe is 9 ft. long and 10 in. in diameter?

14. Find the volume of a cylinder whose altitude is 8 ft., and the diameter of whose base is 2 ft.

LESSON 3

1. If goods are bought 25% below their value, and an allowance of 5% is made for cash payment, what profit will be made if the goods are sold 10% above their value?

2. I have a room 18 ft. by 15 ft. If I cover it with carpet $\frac{3}{4}$ yd. wide, laying the strips lengthwise, I must buy 42 yd.; but if I lay the strips crosswise, only 40 yd. will be needed. Explain by a diagram why this is true.

3. How many *fourth feet* in $1\frac{1}{2}$ rd.? $\frac{1}{4}$ ft. is what part of a rod? What per cent of 2 rd.?

4. A sector of 45° of a circle is what per cent of a circle?

5. If 54 bbl. of apples cost $\$60\frac{3}{4}$, how many barrels can be bought for \$486?

6. Mr. Downs has a field containing 7.5 A. If its length is 242 yd., what is its width?

7. Find the cost of $8\frac{1}{4}$ lb. of coffee at $2\frac{3}{16}$ ¢ an ounce.

8. $133\frac{1}{3}\%$ of \$24 is $\frac{2}{9}$ of the cost of a mowing machine. Find the cost of the machine.

9. Divide 3.375 by 2812.5.

10. An agent's commission for November was \$225, which was 20% less than his commission for October. Find his commission for October.

11. A cube $\frac{1}{2}$ inch long is what per cent of a cube 1 inch long? 2 inches long?

12. What is the ratio of $83\frac{1}{3}$ ¢ to \$1?

13. A boy gave his brother 16 marbles, which was $\frac{2}{7}$ of twice as many as he had left. How many had he at first?

14. Julia is 18 years old, and her age is $\frac{2}{7}$ of 3 times Mildred's age. How old is Mildred?

15. What per cent of a gallon is $1\frac{1}{2}$ qt.?

16. What is the ratio of 3 pence to a shilling?

17. What is the ratio of 18 shillings to a pound?

18. A farmer sold $37\frac{1}{2}\%$ of his corn at 55 ¢ a bushel, and received $\$796\frac{2}{5}$. How many bushels did he have at first?

LESSON 4

1. Five times a certain fraction minus $2\frac{1}{3}$ equals $\frac{2}{3}$. What is the fraction?

2. At $2\frac{3}{4}$¢ each how much will $4\frac{2}{3}$ doz. eggs cost?

3. If $3\frac{1}{2}$ yd. of silk cost \$9.73, how much will 14.7 yd. cost?

4. A miller invested $\$76\frac{1}{8}$ in an equal number of bushels of oats, wheat, and corn, paying $\$\frac{2}{5}$ per bushel for the oats, $\$\frac{7}{8}$ for the wheat, and $\$\frac{9}{10}$ for the corn. How many bushels of each did he get?

5. In a school there are 175 male and 350 female pupils. What per cent of the whole number is each?

6. If 42 qt. of water are mixed with 84 qt. of milk, what per cent of the mixture is each?

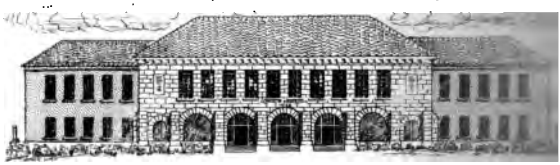
7. A train runs 20 miles in 25 min., and another runs 12 miles in 15 min. Find the ratio of the speed of the second to that of the first.

8. A wheel makes 300 revolutions in $4\frac{2}{7}$ seconds; another makes 150 revolutions in $1\frac{2}{3}$ seconds. Find the ratio of the speed of the first wheel to that of the second.

9. How many board feet are there in 10 pieces of timber 12 in. by 16 in. and 20 ft. long?

10. What per cent is gained on goods bought for 20% less than their value and sold for 20% more than their value?

11. A bin is 6 ft. square. If the bottom is covered with grain to the depth of 15 in., how many bushels are in the bin?



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LESSON 6

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nd describe a parallelogram containing 144

12. The premium paid for insuring goods for $\frac{2}{3}$ of their value, at $1\frac{3}{4}\%$, was \$1365. Find the value of the goods.

13. Find the face of a note which, when discounted at the bank for 9 mo. 27 da. at 8%, will yield \$3582 proceeds.

LESSON 5

1. Two brothers bought 45 bu. of potatoes, the one paying $\$16\frac{1}{2}$, and the other \$6. How many bushels should each get?

2. Find the volume of a square pyramid whose altitude is 8 ft. and the side of whose base is 5 ft.

3. What is the convex surface of a cone whose slant height is 16 ft. and the diameter of whose base is 8 ft.?

4. If a man walks 3 mi. 146 rd. 3 yd. 2.88 in. in 1 hr. 15 min., how long will it take him to walk 2 degrees near the equator?

5. A tank 10 ft. long and 7 ft. wide contains $15\frac{5}{9}$ cu. yd. of water. Find its depth.

6. Find the cost of a draft for \$3040, exchange being at $\frac{3}{4}\%$ discount.

7. When it is 10 A.M. at the Hawaiian Islands, 155° west longitude, what is the time at Washington, 77° 1′ west longitude?

8. The difference in time between the Hawaiian Islands and Cincinnati is 4 hr. 42 min. 24 sec. Find the longitude of Cincinnati.

9. How many boards 15 ft. long will be required to build a fence 5 boards high around a rectangular field 120 rd. long and 30 rd. wide?

10. Find the cash value of a bill of goods amounting to \$12861.80 at 20% discount and 5% off for cash.

11. A man sold two farms for \$2400 each. On one he gained 20% and on the other he lost 20%. Did he gain or lose on the sale, and how much?

12. How many board feet are there in a board which is 20 ft. long, 18 in. wide at one end, and tapers regularly until it is 12 in. wide at the other end?

13. Find the cost, at \$35 per M, board measure, of 10 pieces of sawed timber, each 20 ft. long, 16 in. wide, 12 in. thick.

LESSON 6

1. What is a rhombus? A board having the shape of a rhombus is 16 in. on each side and 8 in. high. Find its area. How does its area compare with that of a square that is 16 in. on each side?

2. A piece of land having the form of a trapezoid contains 47 acres. One of its parallel sides is 65 rd., and the other 95 rd. Find the width of the trapezoid.

3. What is the ratio of a cubic foot to a bushel?

4. If the difference in longitude between A and B is 46° 18′ 45″, what is the difference in time? If A is west of B, what time is it at B when it is 2 o'clock P.M. at A?

5. Divide \$8680 into three parts which shall be to each other as 3, 4, and 6.

6. Find the area of a regular hexagon each side of which is 6 ft., the distance from its center to the middle point of each side being 6 ft.

7. Draw and describe a parallelogram containing 144 sq. ft.

8. Draw and describe a rhomboid containing 96 sq. yd.

9. If I buy potatoes at the rate of 8 bu. for $\$3\frac{1}{5}$ and sell them at the rate of 9 bu. for $\$4.50$, how many bushels must I sell to gain $\$10.50$?

10. A dealer bought a lot of tinware for $\frac{12}{16}$ of its value, and sold it for $\frac{11}{12}$ of its value. If his loss was $\$32\frac{1}{2}$, how much did he pay for it?

11. How many cakes of ice 2 ft. long, 18 in. wide, can be cut from a rectangular pond 360 ft. long, 240 ft. wide, making no allowance for waste?

12. For how much must flour which cost $\$7.50$ a barrel be sold to gain $16\frac{2}{3}\%$, after deducting a commission of 5%?

13. Mr. Day's asking price is 30% above cost. If he sells for 30% of his asking price less, does he gain or lose, and how much?

PARTNERSHIP

LESSON 7

Partnership is the association of individuals for the purpose of carrying on a certain branch of business. The persons constituting the association are called *partners*, and the association thus formed a *house*, *firm*, or *company*.

The money or property put into the business by the partners is called the **Capital** of the company.

The **Resources** or **Assets** of a firm embrace its property of all kinds, its cash on hand, and the amounts due it. Its **Liabilities** are its debts.

For convenience of treatment, partnership is sometimes separated into *simple* and *compound*.

In **Simple Partnership** each partner has his capital invested for the same time.

1. A and B engage in the hardware business. A puts in $6000, and B $4000. They gain $1250. What is each one's share of the gain?

OPERATION

$6000 + $4000 = $10,000 = capital invested by both.

$\frac{6000}{10000}$, or $\frac{3}{5}$ = A's share of the capital.

$\frac{4000}{10000}$, or $\frac{2}{5}$ = B's share of the capital.

Hence, $\frac{3}{5} \times \$1250$, or $750 = A's share of the gain.

and $\frac{2}{5} \times \$1250$, or $500 = B's share of the gain. } *Ans.*

2. A and B enter into partnership to carry on a retail grocery business. A invests $2500 and B $2000. They gain $675. What is each one's share of the gain?

3. Three men, A, B, and C, form a partnership to deal in oysters. A puts in $450, B $575, and C $740. They gain $353. Find each one's share of the gain.

4. A, B, and C engage in the coal business. A contributes $8500, B $9000, and C $12,000. They gain $13,275. Find each one's share of the gain.

5. Three persons, A, B, and C, form a partnership to deal in lumber, with a joint capital of $22,400. At the end of a year they divide the profits, of which A's share is $2250, B's $1950, and C's $2520. How much capital had each invested?

LESSON 8

In **Compound Partnership** each partner's capital is not always employed for the same period of time.

1. A and B enter into partnership to deal in potatoes. A puts in $8000 for 3 yr., and B puts in $6500 for 2 yr. They gain $2960. What is the share of each?

A's $8000 invested for 3 yr. is equivalent to 3 times $8000, or $24,000, for 1 yr. B's $6500 invested for 2 yr. is equivalent to 2 times $6500, or $13,000 for 1 yr. They both have invested equivalent to the sum of $24,000 and $13,000, or $37,000 for 1 yr.

OPERATION

$8000 for 3 yr. = $24000 for 1 yr. = A's.
$6500 for 2 yr. = $13000 for 1 yr. = B's.
$37000 = both for 1 yr.

$\frac{24000}{37000}$, or $\frac{24}{37} \times \$2960 = \$1920$ = A's gain.
$\frac{13000}{37000}$, or $\frac{13}{37} \times \$2960 = \$1040$ = B's gain. } *Ans.*

2. A, B, and C form a partnership. A puts in $3000 for 1 yr., B puts in $2500 for 8 mo., and C $1500 for 6 mo. At the end of the year they divide $2470. Find the share of each.

3. Smith and Jones enter into partnership, Smith contributing $3600, and Jones $1800. At the end of 8 mo. Jones puts in $600 more. At the end of the year they divide $1680. Find each man's share of the gain.

4. Jan. 1, 1901, Mr. Samson commenced dealing in coal with a capital of $5000. May 1, Mr. Straw became a partner with a capital of $3000. Sept. 1, Mr. Riddle put in $4000. Jan. 1, 1902, they found their loss to be $1250. How much of the loss should each man bear?

5. Wilson, Allen, and Thompson entered into partnership April 1, 1900. Wilson put in $2500, Allen $3000, and Thompson $3500. At the end of one year they each withdrew $1000 from the business. April 1, 1902, they found their loss to be $1050. Find what part of the loss each should bear.

6. Anson and Roll entered into partnership to continue for $1\frac{1}{2}$ yr. During the first 9 mo. Anson's capital was $\frac{2}{3}$ as much as Roll's, and during the last 9 mo. $\frac{5}{6}$ as much as Roll's. They gained $13,230. What was the share of each?

7. Payne, Collins, and Arnold formed a partnership for speculative purposes. Payne invested $1600 for 9 mo., Collins $1200 for 1 yr. 1 mo., and Arnold $2400 long enough to entitle him to receive $1456 profit. Find the time Arnold's capital was invested, and the shares of Payne and Collins respectively, the entire gain being $2756.

COMPOUND INTEREST

LESSON 9

If I borrow $1000 for a year, at 6 %, at the end of the year I owe $1000, plus the interest ($60) for one year, or $1060. If I desire to keep the money longer, and do not pay the interest due at the end of the first year, I must pay interest on $1060. The interest for the second year would therefore be $63.60. If I desire to keep the money still longer, and do not pay the interest due at the end of the second year, I must pay interest on $1123.60, etc. This plan of computing interest is based on the assumption that if interest is not paid when due, it should bear interest until paid. The plan of adding the interest to the principal, and thus forming a new principal, at regular intervals of time, is called *compounding interest.*

Compound Interest, therefore, is interest computed on both principal and interest, if the interest is not paid when due.

Simple Interest is computed on the principal alone.

It is customary to add the simple interest annually, semiannually, or quarterly, as the parties may agree. Unless otherwise specified interest is to be added annually.

1. What is the compound interest of $600 for 3 yr. at 6%?

OPERATION I

$600 = principal.
.06
36.00 = int. due at the end of first year.
$600.
$636. = 2d prin., or amt. due at the end of first year.
.06
38.16 = int. due at the end of second year.
$636.
$674.16 = 3d prin., or amt. due at the end of second year.
.06
40.4496 = int. due at the end of third year.
$674.16
$714.6096 = 4th prin., or amt. due at the end of third year.
$600. = 1st prin. to be deducted.
$114.6096 = compound interest. *Ans.*

OPERATION II

$600 = principal.
1.06
3600
600
$636.00 = amt. due at the end of first year.
1.06
3816
636
$674.16 = amt. due at the end of second year.
1.06
404496
67416
$714.6096 = amt. due at the end of third year.
$600
$114.6096 = compound interest. *Ans.*

STATEMENT I

($600 × 1.06 × 1.06 × 1.06) − $600 = answer.

STATEMENT II

$600 × $(1.06)^3$ − $600 = answer.

The work may be very much abbreviated by getting from the Compound Interest Table on page 19 the amount of $1 for the given time and rate. Thus, the amount of $1 for 3 yr. at 6 % is $1.191016; the amount of $600 is 600 times $1.191016, or $714.6096.

STATEMENT III

($1.191016 × 600) − $600 = answer.

NOTE. — When the interest is payable semiannually or quarterly, we find the interest for a half year or a quarter of a year, to which we add the principal, and thus form a new principal every such period.

2. Find the compound int. of $560 for 3 yr. at 6%.

3. Find the compound int. of $850 for 4 yr. at 6%.

4. Find the amount of $4760 for 2 yr. compounded semiannually at 8%.

NOTE. — Multiply the principal by one half the annual rate to find the int. for a half year, and by one fourth the annual rate to find the int. for a quarter of a year.

5. Find the amount of $4780 for 2 yr. 6 mo. compounded semiannually at 4%.

6. Find the interest of $880 for 1 yr. 6 mo. compounded quarterly at 8%.

7. Find the compound interest of $960 for 1 yr. 4 mo. 21 da. at 4%, interest payable quarterly.

NOTE. — If the time consists of years, months, and days, we find the amount for the entire number of years, and then the amount of the last amount for the remaining time.

Compound Interest Table

Showing the amount of $1, at 2, 2½, 3, 3½, 4, 4½, 5, 6, 7, 8, 9, and 10 % compound interest, from 1 to 25 years.

Yr.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.
1	1.0200 0000	1.0250 0000	1.0300 0000	1.0350 0000	1.0400 0000	1.0450 0000
2	1.0404 0000	1.0506 2500	1.0609 0000	1.0712 2500	1.0816 0000	1.0920 2500
3	1.0612 0800	1.0768 9062	1.0927 2700	1.1087 1787	1.1248 6400	1.1411 6612
4	1.0824 3216	1.1038 1289	1.1255 0881	1.1475 2300	1.1698 5856	1.1925 1860
5	1.1040 8080	1.1314 0821	1.1592 7407	1.1876 8631	1.2166 5290	1.2461 8194
6	1.1261 6242	1.1596 9342	1.1940 5230	1.2292 5533	1.2653 1902	1.3022 6012
7	1.1486 8567	1.1886 8575	1.2298 7387	1.2722 7926	1.3159 3178	1.3608 6183
8	1.1716 5938	1.2184 0290	1.2667 7008	1.3168 0904	1.3685 6905	1.4221 0061
9	1.1950 9257	1.2488 6297	1.3047 7318	1.3628 9735	1.4233 1181	1.4860 9514
10	1.2189 9442	1.2800 8454	1.3439 1638	1.4105 9876	1.4802 4428	1.5529 6942
11	1.2433 7431	1.3120 8666	1.3842 3387	1.4599 6972	1.5394 5406	1.6228 5305
12	1.2682 4179	1.3448 8882	1.4257 6089	1.5110 6866	1.6010 3222	1.6958 8143
13	1.2936 0663	1.3785 1104	1.4685 3371	1.5639 5606	1.6650 7851	1.7721 9610
14	1.3194 7876	1.4129 7382	1.5125 8972	1.6186 9452	1.7316 7645	1.8519 4492
15	1.3458 6834	1.4482 9817	1.5579 6742	1.6753 4883	1.8009 4351	1.9352 8244
16	1.3727 8570	1.4845 0562	1.6047 0644	1.7339 8604	1.8729 8125	2.0223 7015
17	1.4002 4142	1.5216 1826	1.6528 4763	1.7946 7555	1.9479 0050	2.1133 7681
18	1.4282 4625	1.5596 5872	1.7024 3306	1.8574 8920	2.0258 1652	2.2084 7877
19	1.4568 1117	1.5986 5019	1.7535 0605	1.9225 0183	2.1068 4918	2.3078 6031
20	1.4859 4740	1.6386 1644	1.8061 1123	1.9897 8886	2.1911 2314	2.4117 1402
21	1.5156 6634	1.6795 8185	1.8602 9457	2.0594 8147	2.2787 6807	2.5202 4116
22	1.5459 7967	1.7215 7140	1.9161 0341	2.1315 1158	2.3699 1879	2.6886 5201
23	1.5768 9926	1.7646 1068	1.9735 8651	2.2061 1448	2.4647 1555	2.7521 7635
24	1.6084 3725	1.8087 2595	2.0327 9411	2.2833 2849	2.5633 0417	2.8760 1388
25	1.6406 0599	1.8539 4410	2.0937 7793	2.3632 4498	2.6658 3633	3.0054 3446

Yr.	5 per cent.	6 per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	1.0500 000	1.0600 000	1.0700 000	1.0800 000	1.0900 000	1.1000 000
2	1.1025 000	1.1236 000	1.1449 000	1.1664 000	1.1881 000	1.2100 000
3	1.1576 250	1.1910 160	1.2250 430	1.2597 120	1.2950 290	1.3310 000
4	1.2155 063	1.2624 770	1.3107 960	1.3604 890	1.4115 816	1.4641 000
5	1.2762 816	1.3382 256	1.4025 517	1.4693 281	1.5386 240	1.6105 100
6	1.3400 956	1.4185 191	1.5007 804	1.5868 743	1.6771 001	1.7715 610
7	1.4071 004	1.5036 303	1.6057 815	1.7138 243	1.8280 891	1.9487 171
8	1.4774 554	1.5938 481	1.7181 862	1.8509 302	1.9925 626	2.1435 888
9	1.5513 282	1.6894 790	1.8384 592	1.9990 046	2.1718 933	2.3579 477
10	1.6288 946	1.7908 477	1.9671 514	2.1589 250	2.3673 637	2.5937 425
11	1.7103 394	1.8982 986	2.1048 520	2.3316 390	2.5804 264	2.8531 167
12	1.7958 563	2.0121 965	2.2521 916	2.5181 701	2.8126 648	3.1384 284
13	1.8856 491	2.1329 283	2.4098 450	2.7196 237	3.0658 046	3.4522 712
14	1.9799 816	2.2609 040	2.5785 342	2.9371 936	3.3417 270	3.7974 983
15	2.0789 282	2.3965 582	2.7590 315	3.1721 691	3.6424 825	4.1772 482
16	2.1828 746	2.5403 517	2.9521 638	3.4259 426	3.9703 059	4.5949 730
17	2.2920 183	2.6927 728	3.1588 152	3.7000 181	4.3276 334	5.0544 703
18	2.4066 192	2.8543 392	3.3799 323	3.9960 195	4.7171 204	5.5599 173
19	2.5269 502	3.0255 995	3.6165 275	4.3157 011	5.1416 613	6.1159 390
20	2.6532 977	3.2071 355	3.8696 845	4.6609 571	5.6044 108	6.7275 000
21	2.7859 626	3.3995 636	4.1405 624	5.0338 337	6.1088 077	7.4002 499
22	2.9252 607	3.6035 374	4.4304 017	5.4365 404	6.6586 004	8.1402 749
23	3.0715 238	3.8197 497	4.7405 299	5.8714 637	7.2578 745	8.9543 024
24	3.2250 999	4.0489 346	5.0723 670	6.3411 807	7.9110 832	9.8497 327
25	3.3863 549	4.2918 707	5.4274 326	6.8484 752	8.6230 807	10.8347 059

PRESENT WORTH AND TRUE DISCOUNT

LESSON 10

1. If you have a note promising to pay you $106 in one year without interest, how much will the note be worth at the end of the year?

2. What sum of money put at interest at 6% for 1 yr. will amount to $106?

3. Which is worth more, a note for $106 due in 1 yr. without interest, or $100 cash, if money is worth 6%?

4. James King bought of Henry Gates a piece of land, giving him a note for $530, due in one year without interest. Following is the note:

$530. ALBANY, N.Y., Dec. 18, 1901.

One year after date, for value received, I promise to pay Henry Gates, or order, Five Hundred and Thirty Dollars.

JAMES KING.

How much will the note be worth when it is due? When is the note due?

5. What sum of money put at interest Dec. 18, 1901, at 6%, will amount to $530 Dec. 18, 1902?

6. How much is the above note worth Dec. 18, 1901?

The **Present Worth** of a sum of money due at some future time, without interest, is that sum which, if put at

interest for the given time, will yield an amount equal to the given sum. Thus, the present worth of $318 due 1 yr. hence, without interest, is $300, money being worth 6%, because $300 put at interest for 1 yr. at 6% will amount to $318.

True Discount is the difference between the amount of the debt bearing no interest and its present worth. In the preceding example the difference between $318 and $300, or $18, is the discount.

7. What is the present worth of $553.80, due 5 yr. hence, without interest, when money is worth 6%?

OPERATION

($1.00 × .06 × 5) + $1.00 = $1.30 = amt. of $1 for 5 yr. at 6%.

$553.80 ÷ $1.30 = 426. $426 present worth. *Ans.*

The amt. of $1 for 5 yr. at 6% is $1.30; hence the present worth of $1.30 is $1, and the present worth of $553.80 is as many dollars as $1.30 is contained times in $553.80, or 426.

8. What is the present worth of a debt of $2688, payable in 2 yr., without interest, when money is worth 6%?

9. What is the present worth of $1500, due in 1 yr. 6 mo., without interest, money being worth 6%?

10. What is the present worth of $257.50, due in 2 yr. 6 mo., without interest, money being worth 9%?

11. What is the true discount of $1280.36, due in 9 mo., without interest, when money is worth $4\frac{1}{2}$%?

12. What is the difference between the true discount and the interest of $3860.40 for 8 mo., money being worth $4\frac{1}{2}$%?

13. Which is more profitable, to buy flour at $\$4\frac{1}{2}$ a barrel on 4 mo., or at $\$5$ a barrel on 6 mo., money being worth 6%?

14. I bought $\$600$ worth of furniture, receiving a discount of 10% for cash, and sold it at list price on 2 mo. time. What was my gain, money being worth 6%?

15. I bought goods to the amount of $\$3000$ on 60 da. If I am offered 2% discount for cash, shall I gain or lose, if I accept the offer, money being worth 6%?

16. A merchant bought goods at three different times, as follows: $\$3860$ worth on 3 mo., $\$580$ worth on 2 mo., $\$975$ worth on 4 mo. How much ready cash will be required to pay the bills, discounting at 6%?

REVIEW WORK

LESSON 11

1. Define partnership. Simple partnership. Compound partnership. Capital. Resources or assets.

2. Three brothers, A, B, and C, engage in business. A invests $4000, B $6000, and C $8000. What is the share of each if the gain is $7200?

3. Mr. Walker had his dry goods insured in one company for $1600 and in another company for $2400. If, in case of a fire, $1000 of his stock is destroyed, what sum must each company pay?

4. What is insurance? Premium? Policy?

5. State the rule for finding the convex surface of a pyramid.

6. Find the convex surface of a square pyramid having a base 6 ft. square and slant height 15 ft.

7. Find the volume of a cylinder whose circumference is 24 in., and whose altitude is 16 in.

8. In what time will $1000 gain $90 at 6%?

9. What principal will amount to $10,800 in 1 yr. 4 mo. at 4%?

10. Find the interest of $750 from Nov. 12, 1901, to Dec. 30, 1902, at $5\frac{1}{2}$%.

11. A, B, and C agreed to do a piece of work for $315. A worked 6 da., B 10 da., and C 14 da. What part should each receive?

12. My agent charged me $24.70 for selling two car loads of potatoes. For how much were the two car loads sold, his commission being 5%?

LESSON 12

$600. WASHINGTON, D. C., Aug. 28, 1901.

On demand, for value received, I promise to pay Charles Bertles, or order, Six Hundred Dollars, with interest at $3\frac{1}{2}\%$.

JAMES LUFT.

1. Find the value of the above note April 1, 1902.

2. Find the number of square feet in a piece of land 6 ch. 50 li. long and 4 ch. 25 li. wide.

3. I own $6\frac{3}{4}$ acres of land which I intend to divide into 27 building lots. How many square rods will each lot contain?

4. A man paid $96 for insuring his factory at 1% on $\frac{3}{4}$ of its value. Find the value of the factory.

5. A is 165° west of B. When it is 10 P.M. at B, what time is it at A?

6. 1582 is 13% more than what number?

7. A merchant sold 600 barrels of flour at a loss of 20%. If he lost $360, how much per barrel did the flour cost him?

8. Find the cost of 288 fence boards, each 16 ft. long and 8 in. wide, at $16 per M.

9. A grocer bought a barrel of molasses ($31\frac{1}{2}$ gal.) at 40¢ a gallon. He retailed it at 12¢ a quart. Find his gain, and gain per cent.

10. Three brothers, A, B, and C, engaged in business. A furnished $\frac{1}{4}$ of the capital, B 35%, and C the remainder. A's capital was in the business 7 mo., B's 6 mo., and C's 5 mo. They gained $561.60. Find each partner's share of the gain.

11. $83\frac{1}{3}\%$ of a number is 36 less than the number. What is the number?

12. If 8 yr. is subtracted from 50% of a man's age, the remainder will equal $33\frac{1}{3}\%$ of his age. What is his age?

13. The school tax of a certain city is $95,640. Find the value of the taxable property if the rate of taxation is 6 mills on the dollar.

14. If we consider the circumference of the earth to be 25,000 mi. find the distance in miles between two places on the equator which are 90° apart.

15. Considering the distance of the earth from the sun to be 93,000,000 mi., and that it takes $8\frac{1}{3}$ min. for a ray of light to pass from the sun to the earth, what is the velocity of light expressed in miles per second?

PARTIAL PAYMENTS

LESSON 13

When partial payments of notes and other interest-bearing obligations are made, the amounts and times of the several payments are written on the back of the obligation as receipts. Such entries on the back of the obligation are called **Indorsements.**

1.

$\$500\frac{00}{100}$. HARRISBURG, PA., Sept. 26, 1897.

Three years after date, for value received, I promise to pay John Edie, or order, Five Hundred Dollars, with interest at 6%.

JONAS LONG.

On this note the following payments were indorsed: Jan. 14, 1898, $25; March 29, 1898, $40; April 25, 1899, $8; Oct. 16, 1899, $12. How much was due Nov. 13, 1900.

OPERATION

Face of note	$500.00
Int. on $500 from Sept. 26, 1897, to Jan. 14, 1898 (3 mo. 18 da.)	
1898 1 14 − 1897 9 26 = 3 18	9.00
Amt. due at 1st payment Jan. 14, 1898	$509.00
First payment to be deducted	25.00
Bal. due Jan. 14, 1898 (1st new principal)	484.00
Int. on $484 from Jan. 14, 1898, to March 29, 1898 (2 mo. 15 da.)	6.05
Amt. due March 29, 1898	490.05

Amt. brought forward	490.05
Second payment to be deducted	40.00
Bal. due March 29, 1898 (2d new principal)	450.05
Int. on $450.05 from March 29, 1898, to April 25, 1899 (1 yr. 26 da.)	28.95
Third payment, $8, being less than the interest due, is not deducted.	
Int. on $450.05 from Apr. 25, 1899, to Oct. 16, 1899 (5 mo. 21 da.)	12.83
The sum of the 3d and 4th payments, $20, being less than the interest due, is not deducted.	
Int. on $450.05 from Oct. 16, 1899, to Nov. 13, 1900 (1 yr. 27 da.)	29.03
Amt. due Nov. 13, 1900	520.86
Sum of 3d and 4th payments to be deducted	20.00
Bal. due Nov. 13, 1900	$500.86

Ans.

When partial payments have been made on interest-bearing obligations running longer than a year, the method employed in the preceding example for computing interest is the one adopted by the United States courts, and is, therefore, known as the *United States rule*. It is based upon the principle that neither payment nor interest shall bear interest. It may be seen in the preceding example that the 3d payment ($8) is less than the interest ($28.95) due, and is not deducted as in the case of the 1st and 2d payments.

If we should add the $28.95 to the 2d new principal ($450.05), and then deduct the 3d payment, as we did in the case of the 1st and 2d payments, there would remain, as 3d new principal, $471. If we then computed interest on $471 up to the next payment, we should be computing *interest* on *interest*, and therefore violating the principle upon which this rule is founded.

When a payment is less than the interest due, we take no account of it until such time as the payments taken together equal or exceed the interest due. We then proceed as we did in the case of the 1st and 2d payments. Accordingly, in the preceding example, the sum of the 3d and 4th payments ($20), being less than the interest due ($41.78) at the time the 4th payment was made, is not deducted. We next find the amount due at the date of settlement, and from this amount deduct the sum of the last two payments.

SUGGESTION TO TEACHER. — Have the pupils study thoroughly the preceding illustrative example. They should be required to verify every step in the operation before attempting to solve the problems in the following lesson.

LESSON 14

1

$8000. DENVER, COL., Oct. 3, 1898.

Three months after date, for value received, I promise to pay J. N. Schott, or order, Eight Thousand Dollars, with interest at 6%. T. S. HAMMER.

On this note the following payments were endorsed: Jan. 1, 1899, $200; March 5, 1899, $100; Sept. 25, 1899, $150; Nov. 23, 1899, $400; Dec. 25, 1899, $250. How much was due Jan. 1, 1900?

2. A note for $5600, dated April 1, 1898, had the following payments indorsed on it: July 1, 1898, $200; Nov. 1, 1898, $75; Jan. 1, 1899, $125; Aug. 12, 1899, $600. How much was due March 31, 1900, interest 6%?

3. A note for $9500, dated Sept. 12, 1895, had indorsed on it the following payments: Nov. 15, 1895, $400; April

1, 1896, $190; Oct. 3, 1899, $450; April 25, 1900, $140. How much was due July 1, 1900, interest 6%?

4. A note for $7500, dated Aug. 26, 1899, had the following payments indorsed on it: Oct. 14, 1899, $250; Jan. 1, 1900, $60; June 30, 1900, $80; Sept. 25, 1900, $650. How much was due Dec. 1, 1900, interest 6%?

Commercial Rule

When notes and accounts bearing interest are settled within a year, the following method of computing interest is usually employed.

5. I had a note for $2000, dated April 1, 1900, on which the following payments were indorsed: June 25, 1900, $450; Aug. 16, 1900, $800; Nov. 10, 1900, $600; Jan. 1, 1901, $150. How much was due April 1, 1901, interest 6%?

Operation

Amount of $2000 for 1 yr. at 6%		$2120.00
Amt. of 1st payt. to time of settlement (9 mo. 6 da.)	$470.70	
Amt. of 2d payt. to time of settlement (7 mo. 15 da.)	830.00	
Amt. of 3d payt. to time of settlement (4 mo. 21 da.)	614.10	
Amt. of 4th payt. to time of settlement (3 mo.) .	152.25	2067.05
Balance due April 1, 1901		$52.95

Ans.

6. A note for $884, dated Jan. 15, 1900, had indorsed on it the following payments: Feb. 25, 1900, $100; May 10, 1900, $288; July 25, 1900, $260; Sept. 19, 1900, $150. Find the balance due Jan. 15, 1901, interest 6%.

7. A note for $6840, dated May 15, 1901, bears indorsements as follows: July 21, 1901, $300; Oct. 10, 1901, $500; Feb. 24, 1902, $200. Find the balance due May 15, 1902, interest 6%.

STOCKS AND BONDS

STOCKS

LESSON 15

It is a very common thing for a number of individuals to associate themselves together for the purpose of inaugurating and conducting certain commercial enterprises.

When persons unite to carry on any particular branch of business, and enter into an agreement by contract, the association is called a **Partnership**, and the individuals are called partners. (See Lesson 7.) But if they effect an organization, that is, elect a president, secretary, treasurer, board of directors, and secure a charter, the association is known as a **Corporation**, or **Stock Company**.

Railroads, banks, manufacturing companies, insurance companies, etc., are illustrations of corporations, or stock companies.

The **Charter** is the certificate given to the corporation by the legislature of the state, or by some state official. It specifies the name of the corporation, and the amount of capital, and defines its powers and obligations.

The amount of money and other property employed to carry on the business of the corporation is called its **Capital**, or **Stock**.

The capital invested is divided into equal shares, usually of $100 each. Sometimes each share represents more than $100 of capital, and frequently less.

Persons who own shares of stock are called **Stockholders.** They constitute the corporation.

Each stockholder receives a paper or document issued by the corporation specifying the number and value of the shares which he owns. Such a document is called a **Certificate of Stock.**

The following is a form of a certificate of stock:

No. 38 20 Shares

FIRWOOD TAPE MANUFACTURING COMPANY

WILKESBARRE, PA., Dec. 26, 1900.

Capital, $50,000
Shares, $100 each

This certifies that HERMAN K. PHILIPS *is entitled to* TWENTY SHARES *in the capital stock of the* FIRWOOD TAPE MANUFACTURING COMPANY, *transferable only on the books of the Company, in person or by attorney, upon the surrender of this certificate.*

In witness whereof the seal of the Company is hereunto affixed.

M. P. SPARE, *Sec'y.* T. F. JOHNSON, *Pres't.*

The **Par Value** of stock is its original, or face value.

1. What is the par value of one share of stock in the Firwood Tape Manufacturing Company? What is the amount of the capital stock? Into how many shares is the stock divided?

2. Who is president of the company? Who is secretary?

3. Who is the owner of this certificate of stock? What is the number of the certificate? To how many shares is Mr. Philips entitled?

4. If the shares were purchased at par, how much did Mr. Philips pay for them?

LESSON 16

A **Dividend** is a sum of money distributed among the stockholders out of the gains of the business.

An **Assessment** is a sum of money required of stockholders to meet losses or expenses.

Dividends and assessments are computed, at a certain rate per cent, on the par value of stock. The profits are divided among the stockholders periodically, that is, quarterly, semiannually, or annually.

A stockholder cannot compel the company to return him the money which he paid for his shares, but he can sell his stock.

Stock is often bought and sold. The price of stock is constantly fluctuating.

The **Market Value** of stock is what it will sell for at a particular time.

A person who buys and sells stocks, usually for others, is called a **Stockbroker**, or **Broker**. The commission charged by brokers for transacting business for others is called **Brokerage**. The rate is usually $\frac{1}{8}\%$, but it is sometimes as high as $\frac{1}{4}\%$. It is always reckoned on the par value of stock.

When stock sells for its original, or face value, it is said to be at par; when it sells for more than its original value, it is above par, or at a premium; and when it sells for less, it is below par, or at a discount.

Stock is usually quoted at a certain per cent of its original value. When the price of stock is quoted at 105, it is above par, or at a premium of 5%; that is, \$100 worth of stock sells for \$105; or \$1 of stock sells for \$1.05. When the price of stock is 95, it is below par, or

at a discount of 5%; that is, $100 worth of stock sells for $95; or $1 of stock sells for $.95.

1. In 1899 the Firwood Tape Company divided $5000 among the stockholders out of the gains of the company. How much was paid on each share? What was the rate of dividend?

2. How much did Mr. Philips receive as his share?

3. If the president of the company, Mr. Johnson, owned 50 shares, what was his share of the dividend?

4. Define: par value, premium, dividend, certificate of stock, charter, corporation, market value.

LESSON 17

The dividend paid to stockholders does not depend on the price which is paid for the shares, but on their par value.

For example, if Thomas Payne and James Stewart each purchased the same number of $100 shares of stock in a company, both would be entitled to the same amount of dividend, although Payne may have paid $120 a share and Stewart only $80.

1. Find the cost of forty $100 shares of railroad stock at 101, brokerage $\frac{1}{4}\%$.

OPERATION

$\$101 + \$\frac{1}{4} = \$101\frac{1}{4}$, entire cost of 1 share.
$\$101\frac{1}{4} \times 40 = \$4050$, entire cost of 40 shares. *Ans.*

2. Find the cost of 45 shares of bank stock, $100, at $103\frac{1}{2}$, brokerage $\frac{1}{8}\%$.

3. Find the cost of 25 shares of railroad stock, at 98, brokerage $\frac{1}{8}\%$.

NOTE. — Brokerage is not allowed in the following examples unless otherwise stated.

4. How many shares, $100, of railroad stock may be bought for $9050, at $113\frac{1}{8}$?

STATEMENT

$9050 ÷ $113\frac{1}{8}$ = answer.

5. A man owns 40 shares of railroad stock, $100, which pays 8% per annum. How much dividend does he receive?

$100 × 40 × .08 = answer. Explain.

6. Mr. Wilson paid $240 per share for each $100 share of stock in a mining company. What per cent does he receive on his investment, when the company pays an annual dividend of 8%?

He receives $8 on an investment of $240.
His per cent of gain = $8 ÷ $240 = $.03\frac{1}{3} = 3\frac{1}{3}\%$. *Ans.*

7. Mr. Ames bought 70 shares, $50, of stock at $62\frac{1}{2}$. If the company pays a dividend of 5%, what per cent does Mr. Ames receive on his investment?

8. A gas company pays an annual dividend of 8%. How much does Mr. B. receive if he owns 80 shares of $50 each? What per cent does Mr. B. receive on his investment if he paid $65 a share?

LESSON 18

1. How many shares of stock, $50, at $\frac{1}{4}\%$ discount, can I buy for $3591?

2. Mr. Carle sent his broker in Philadelphia $6390 to invest in railroad stock at $53\frac{1}{8}$. How many $50 shares did he get, brokerage $\frac{1}{4}\%$?

3. My broker sold for me 150 shares railroad stock, \$50, at $55\frac{1}{2}$, and invested the proceeds in mining stock, \$100, at $\$104\frac{1}{2}$, brokerage $\frac{1}{4}\%$ in each case. How many shares did I get, and how much surplus?

4. I received \$301 as my annual dividend from a $3\frac{1}{2}\%$ stock. How much stock do I hold?

OPERATION

$\$301 \div .03\frac{1}{2} = \$8600$. *Ans.*

Since \$1 of stock yields $\$.03\frac{1}{2}$ income, \$301 must be $.03\frac{1}{2}$ of the value of the stock.

5. Mr. Harvey received \$225 as his quarterly dividend from a 10% stock. Find the par value of the stock.

6. A railroad company divided \$44,800 among its stockholders as an 8% dividend. Find the face of the stock. How many \$100 shares do I own if my share of the dividend was \$4000?

7. What sum must I invest in 5% stock, at 103, to yield an income of \$480?

OPERATION

$\$480 \div .05 = \$9600$, par value of stock.

$\$1.03 \times 9600 = \$9888$, sum invested. *Ans.*

Since \$1 of stock yields \$.05 income, \$480 must be .05 of the par value of the stock. Hence the par value must equal as many dollars as .05 is contained times in \$480, or \$9600. Since the cost of \$1 of stock is \$1.03, the cost of \$9600 of stock is 9600 times \$1.03, which equals \$9888.

STATEMENT

$\$1.03 \times (480 \div .05) =$ answer.

8. What will be my annual income if I invest \$3120 in a 5% stock, selling at 104?

OPERATION

$\$3120 \div 1.04 = \$3000$, par value of stock.

$\$3000 \times .05 = \$150$ income. *Ans.*

Since $1.04 is paid for $1 of stock, for $3120 there can be bought as many dollars of stock as $1.04 is contained times in $3120, or $3000. 5% of $3000 = $150.

STATEMENT

($3120 ÷ 1.04) × .05 = answer.

9. What income will be derived from $9785 invested in a $5\frac{1}{2}\%$ stock selling at 103?

10. What income will be derived from $7225 invested in a $4\frac{1}{2}\%$ stock selling at 85?

11. If I buy 5% stock at 90, what rate of interest do I receive on my investment?

OPERATION

$5 ÷ $90 = $.05\frac{5}{9} = 5\frac{5}{9}\%$. *Ans.*

Since each share costs $90 and yields an income of $5, the rate of income on the investment equals $5 ÷ $90, or $.05\frac{5}{9} = 5\frac{5}{9}\%$.

BONDS

LESSON 19

Corporations, like private individuals, often need money to meet exceptional expenditure. This money they borrow, giving a mortgage on their property as security for its payment. These mortgages are called *bonds*. A **Bond** is a kind of promissory note securing the payment of a sum of money at or before a specified time.

Bonds bear a fixed rate of interest payable at certain definite periods, as quarterly, semiannually, or annually.

If you invest money in bonds, you are entitled to a specified income; while if you invest in stocks, you receive only your share of the profits after all expenses, including interest on bonds, have been paid.

Bonds are issued not only by private corporations, but also by the United States government, states, counties, cities, towns, etc.

Bonds derive their name chiefly from the corporation issuing them, the rate of interest they bear, and the date at which they are due.

Take for example the U. S. 4's, 1907. They were issued by the United States government, they bear 4% interest, and are due in 1907. Again, we have U. S. 3's, 1918. They were issued by the government in 1898 to raise money to prosecute the war with Spain. They are payable Aug. 1, 1918, twenty years after date of their issue, with interest at 3% per annum, payable quarterly on the first day of November, February, May, and August.

Government bonds have attached to them small certificates of interest called **Coupons.**

These coupons are cut off quarterly, semiannually, or annually, as they become due, and presented at bank or some other depository for payment.

Bonds, like stocks, are bought and sold at a special market called a **Stock Exchange.** The price fluctuates according to the demand, or as money is plentiful or scarce. Many people wish to invest their money in government bonds because they are so secure. For this reason they always command a premium.

1. When bonds are at 3% premium, what is a $100 bond worth? What is $1 of bond worth? When bonds sell for a lower price than their face value, they are said to be below par or at a discount.

2. When bonds sell at a discount of 3%, what is a $100 bond worth? What is $1 of bond worth?

3. When bonds are at a premium of 5%, how much will ten $500 bonds cost?

4. Find the cost of twenty $100 bonds at a discount of 5%.

LESSON 20

1. What annual income will be realized from $10,000 worth of 4½% bonds purchased at par?

2. Find the cost of five $1000 U. S. bonds at 105½, brokerage ⅛%. (Brokerage is reckoned on the par value.)

3. When bonds are selling at a premium of 2%, how many $100 bonds can be bought for $7650?

SUGGESTION. — A $100 bond will cost $102. Why?

4. At 3½% premium, how many $100 bonds may be bought for $4140?

5. How much must be invested in United States 4% bonds to yield an annual income of $1600?

OPERATION

$1600 ÷ .04 = $40,000, sum invested. *Ans.*

Since $1 of bond yields $.04 income, $1600 must be .04 of the sum to be invested.

6. What sum must be invested in 3% bonds to secure an annual income of $1080?

7. What sum of money must be invested in U. S. 4½% bonds, at 105, to yield an annual income of $360?

OPERATION

$360 ÷ .04½ = $8000, par value of bonds.
$1.05 × 8000 = $8400, sum invested. *Ans.*

Since $1 of bond yields $.04½ income, $360 must be .04½ of the par value of the bonds. Hence the par value must equal as many dollars

as .04½ is contained times in $360, or $8000. Since the cost of $1 of bond is $1.05, the cost of $8000 of bond is 8000 times $1.05, or $8400.

8. What sum of money must be invested in 5% city school bonds, selling at 101¾, to yield an annual income of $1800?

9. What are bonds? How do they differ from stocks? What are coupons?

LESSON 21

1. When United States 6% bonds are bought at 120, what rate per cent of income is realized?

OPERATION

$6 ÷ $120 = .05 = 5%. *Ans.*

Since each share costs $120, and yields an income of $6, the rate of income on the investment equals $6 ÷ $120, or .05 = 5%.

2. When United States 6% bonds are bought at 112, what rate of income is realized?

3. When stock yielding 9% is bought at a premium of 30%, what rate of income is realized?

4. A man bought bank stock which pays 5% semi-annually, at a premium of 40%. What rate of income did he realize on the investment?

5. Stock paying 5% dividend is purchased at 10% discount. What rate per cent is realized on the investment?

6. What must be paid for a 5% stock in order to realize 6% on the investment?

One share, $100, of 5% stock yields an income of $5; hence $5 must equal .06 of the price to be paid. $5 ÷ .06 = 83⅓. *Ans.*

7. What must be paid for a 4% stock to realize 5% on the investment?

8. What price must be paid for 8% stock to realize an income of 6%?

9. What must be paid for stock yielding 10% to realize 8% on the investment?

10. A man bought railroad stock at a discount of $3\frac{1}{4}\%$, and sold it at a premium of $4\frac{3}{4}\%$, thereby gaining \$640. Find the par value of the stock.

\$1 of stock was bought for $100\% - 3\frac{1}{4}\%$, or $96\frac{3}{4}\%$ of its par value, and sold for $100\% + 4\frac{3}{4}\%$, or $104\frac{3}{4}\%$ of its par value. The gain on \$1 of stock $= 104\frac{3}{4}\% - 96\frac{3}{4}\%$, or 8% of its par value. Hence the par value of the stock = \$640 ÷ .08, or \$8000. *Ans.*

11. I paid my broker \$25 for selling Jersey Central stock, \$100, brokerage $\frac{1}{8}\%$. How many shares did he sell?

12. I paid my broker \$75 for selling Wyoming Valley Traction Company stock, \$100, brokerage $\frac{1}{4}\%$. Find the par value of the stock, and the number of shares sold.

LESSON 22

1. My broker bought for me 50 shares of bank stock, \$100, at 125, and sold them at $130\frac{1}{2}\%$. What was my profit, brokerage $\frac{1}{4}\%$?

2. A man bought 60 shares of D. L. and W. stock, \$100, at 107, and after receiving an 8% dividend sold them at $102\frac{1}{2}$, brokerage $\frac{1}{4}\%$. Did he gain or lose, interest on the money invested not considered?

3. If a man received \$1325 as a 5% dividend on stock bought at 107, how much had he invested?

4. What per cent dividend is declared if I receive $600 on $11,420 invested in bank stock at $142\frac{3}{4}$?

5. What per cent dividend is declared if I receive $382.50 on $8372.50 invested in turnpike stock at $98\frac{1}{2}$?

6. I own 300 shares of stock, $100. July 1, I received a stock dividend of 15 shares; and Jan. 1, I received another of 18 shares and $90 in money. Find the rates of dividend declared.

7. Johnson rents his farm for $720 a year, which is 6% of its value, and pays a tax of 2 mills on its valuation, and an annual insurance of $36. If he sells his farm and invests in a 5% stock at 90, will his annual income be increased or diminished, and how much?

8. Which is the better investment, $4\frac{1}{2}$% stock bought at 90, or $5\frac{1}{2}$% stock at 110?

9. Which is the better investment, stock yielding 6% dividend, bought at 20% discount, or stock yielding 8% bought at 90?

10. What price must be paid for 5% stock so that it will yield the same income as 10% stock bought at 160?

REVIEW WORK

LESSON 23

1. Which is greater, and how much, the perimeter of a square whose side is 12 ft., or the circumference of a circle whose radius is 8 ft.?

2. Compare the volume of a square prism whose side is 5 ft. and altitude 9 ft. with that of a square pyramid having the same base and altitude.

3. If 36 men can build a wall 240 ft. long, $3\frac{1}{3}$ yd. high, and 3 ft. thick in 30 da. of 10 hr. each, in how many days of 9 hr. each will 18 men build a wall 120 ft. long, 3 yd. high, and 5 ft. thick?

4. I bought 40 acres of land for $1500, and sold it at $45 an acre. What was the gain per cent?

5. How many feet long is a chain? How many yards? How many rods?

6. How many chains are there in 1221 ft.?

7. How many chains are there in 3168 in.?

8. How many board feet are there in 20 planks 16 ft. long, 18 in. wide, and 2 in. thick?

9. If the cost of mowing 7.75 acres of grass is $15.50, at the same rate find the cost of mowing $8\frac{3}{8}$ acres.

10. Find the number of feet of lumber in a plank 12 ft. long, 12 in. wide at one end, 10 in. wide at the other, and 3 in. thick.

11. How many cubic inches of water are in a pipe 4 in. in diameter and 16 ft. long?

12. Find the weight of the water in a pipe 20 in. in diameter and 14 ft. long, allowing a cubic foot of water to weigh $62\frac{1}{2}$ lb.

13. If a wheel revolves 40 times in going 500 ft., how many times will it revolve in going $1\frac{3}{4}$ mi.?

LESSON 24

1. A carload of potatoes was bought at 40¢ a bushel, and sold at a loss of $12\frac{1}{2}\%$. If the loss was \$21.60, how many bushels were in the car?

2. Find the value, at \$75 an acre, of a quadrilateral field whose parallel sides are 60 rd. and 40 rd. respectively, and whose width is 32 rd.

3. $9\frac{1}{2}$ is $237\frac{1}{2}\%$ of what number?

4. What is a bank? How may one borrow money at a bank?

5. What is bank discount?

6. If a man wishes to use exactly \$500 for 90 da., for what sum must he write his note, if the bank discount is $4\frac{1}{2}\%$?

7. A 4-mo. note, dated July 24, was discounted Oct. 27, at 6%. If the proceeds were \$2398, for what sum was the note written?

8. What amount of bonds at $97\frac{3}{8}$ can be bought for \$15,580?

9. Mr. James sold 80 U. S. 4's at $112\frac{3}{4}$, brokerage $\frac{1}{8}\%$. How much did he receive for them?

10. What are bonds? What is a corporation or stock company?

11. Define: dividend, assessment, capital.

12. Find the cost of 5 bu. 3 pk. 6 qt. of chestnuts at $3 a bushel.

13. Find the date at which $468, if put at simple interest at 6%, Oct. 20, 1901, will amount to $702.

14. Find the trade discount on a bill of goods for $3000 with 10% and 5% off.

15. If by selling an article for $19 I lose 5%, for how much should I sell it to gain 5%?

16. Find the rate at which $850 in 3 mo. will gain $17.

LESSON 25

1. Define: percentage, base, rate per cent, amount, difference.

2. Write an example in which the base and rate per cent are given to find the percentage.

3. Write an example in which the base and the percentage are given to find the rate per cent.

4. Write an example in which the percentage and rate per cent are given to find the base.

5. Solve the examples you have written and give rule.

6. Find the sum of $2\frac{3}{4}$ doz., 3 score and 10, $\frac{5}{6}$ of a gross, $1\frac{5}{6}$ great gross.

7. Puckey Bros. bought 12 gross of Spencerian engrossing steel pens for $9. If they sell 3 pens for 5¢, how much will they gain, allowing 1 doz. as worthless?

8. A merchant commenced business with a capital of $3000. The first year he gained $33\frac{1}{3}\%$, which he invested in his business. The second year he gained 25%, which he also invested in his business. The third year he lost 15%. What was his annual average gain?

9. Divide 96 into two such parts that the second shall be 6 more than the first.

10. Find the compound interest of $360 at 6% from Jan. 1, 1901, to June 12, 1902, interest semiannually.

11. A man invested $6315 in bonds at 105, and after receiving a 4% dividend, he sold them at 115. How much did he gain, brokerage in each case $\frac{1}{4}\%$?

12. What is a decimal fraction? What distinction is usually made between a decimal fraction and a decimal? ("Fourth Year," p. 97.)

13. Show by an example how to change a common fraction to a decimal; a decimal to a common fraction.

14. State rule for multiplying decimals; for dividing decimals.

ALGEBRA

LESSON 26

Algebra is a branch of mathematics in which some of the quantities considered are represented by letters.

An **Equation** is the expression of equality between two quantities or sets of quantities.

Thus, $9 + 7 = 16$ and $x + 2x = 21$ are equations.

The signs $+$, $-$, $\times$, $\div$, and $=$ have the same meaning and use in this chapter as in arithmetic.

a, b, x, etc., are used for $1a$, $1b$, $1x$.

1. By the use of the signs $+$ and $=$ express the sum of x and $3x$.

MODEL. $x + 3x = 4x$.

In a similar manner express the sum of $2a$ and $3a$; $5b$ and $7b$; $8x$ and $4x$.

2. By the use of the signs $-$ and $=$ express the difference between $5x$ and $3x$.

MODEL. $5x - 3x = 2x$.

In the same way express the difference between $9a$ and $3a$; $7b$ and $3b$; $5y$ and $2y$; $8x$ and $5x$.

3. If x denotes the cost of 1 bushel of potatoes, how many times x will denote the cost of 5 bushels? 8 bushels?

4. If $6x$ denotes the cost of 6 bushels of potatoes, how many times x will denote the cost of 1 bushel? of 3 bushels?

5. If $5a = 25$, what is the value of a?

MODEL. If $5a = 25$, a equals $\frac{1}{5}$ of 25, or 5.

6. If $3x = 21$, what is the value of x? $4x$? $7x$? $9x$?

7. If $5x + 4x = 18$, what is the value of x? $5x$? $7x$? $8x$?

8. $3x + 2x + 4x = 27$. What is the value of x? $6x$? $11x$? $12x$?

9. $7b - 3b = 24$. What is the value of b? $5b$? $7b$? $3b$?

10. $3x + 4x - 2x = 30$. What is the value of x? $7x$?

11. If $5x$ represents John's age and $2x$ represents Henry's age, how many times x will represent the sum of their ages? How many times x will represent the difference of their ages?

12. In the preceding example, if x denotes 4 years, what is John's age? What is Henry's age? What is the value of $5x + 2x$? Of $5x - 2x$?

LESSON 27

1. James and John have 21 marbles. How many has each if John has twice as many as James?

We let x represent the number of marbles which James has; then, since John has twice as many as James, $2x$ will represent John's number, and $x + 2x$, or $3x$, the number both have, or 21. If $3x$ is equal to 21, x is equal to 7, or James's number, and $2x$, or 14, equals John's number.

OPERATION

Let x = James's number,
then $2x$ = John's number,
and $x + 2x = 21$,
$3x = 21$,
$x = 7$, James's number,
$2x = 14$, John's number. } *Ans.*

2. Willis and John picked 36 bushels of apples. How many did each pick if Willis picked twice as many as John?

3. The sum of two numbers is 60. The larger is 3 times the smaller. What are the numbers?

OPERATION

Let x = the smaller number,
then $3x$ = the larger number,
and $x + 3x = 60$, the sum;
$4x = 60$, four times the smaller number,
$x = 15$, the smaller number,
$3x = 45$, the larger number. } *Ans.*

4. Divide 80 cents between Robert and Elizabeth so that Robert shall have 3 times as many cents as Elizabeth.

5. A man bought a horse and carriage for $250. He paid 4 times as much for the horse as for the carriage. Find the cost of each.

6. Divide 108 into two parts so that the larger part shall equal 5 times the smaller.

LESSON 28

1. The sum of three numbers is 96. The second is twice the first, and the third 3 times the first. What are the numbers?

OPERATION

Let x = the first number,
then $2x$ = the second number,
and $3x$ = the third number,
$x + 2x + 3x = 96$, the sum;
$6x = 96$, six times the first number,
$x = 16$, the first number,
$2x = 32$, the second number,
$3x = 48$, the third number. } *Ans.*

2. A man bought a hat, a pair of shoes, and an overcoat for $33. He paid 2 times as much for the shoes as for the hat, and 8 times as much for the overcoat as for the hat. How much did he pay for each?

3. $38,000 is to be divided among three persons. The second is to receive 3 times as much as the first, and the third 5 times as much as the first. Find the share of each.

4. What number added to 3 and 4 times itself will give 72?

5. What number added to 4 and 7 times itself will give 108?

6. The difference of two numbers is 42, and the larger is 3 times the smaller. Find the numbers.

OPERATION

Let x = the smaller number,
then $3x$ = the larger number,
and $3x - x = 42$, the difference;
$2x = 42$, twice the smaller number,
$x = 21$, the smaller number,
$3x = 63$, the larger number. } *Ans.*

7. The difference of two numbers is 72, and the larger number is 4 times the smaller. Find the numbers.

8. A farmer raised 384 bushels more of wheat than of oats. How many bushels of each did he raise if 4 times the number of bushels of oats equals the number of bushels of wheat?

9. A man bought a pair of horses and a carriage for $425, paying 4 times as much for the horses as for the carriage. Find the cost of each.

LESSON 29

Find the value of x, y, z, etc., in the following equations:

1. $x + 2x = 72$.
2. $2x + 3x = 180$.
3. $5y + 2y = 196$.
4. $3z + 4z = 161$.
5. $8y - 3y = 35$.
6. $7w - 4w = 81$.
7. $x + 2x + 3x = 144$.
8. $2x + 4x + 5x = 110$.
9. $3y + 5y + 6y = 294$.
10. $5z + 3z - 4z = 48$.
11. $12w - 3w + 6w = 120$.
12. $23z - 14z - 3z = 192$.

13. A man bought an equal number of sheep and calves for \$270, paying \$4 apiece for the sheep and \$5 apiece for the calves. How many of each did he buy?

14. The difference between 2 times a certain number and 7 times the same number is 70. Find the number.

15. I bought an equal number of pounds of sugar and rice for 96 cents, paying 5 cents a pound for the sugar, and 7 cents for the rice. How many pounds of each did I buy?

16. A man bought an equal number of pigs, calves, and cows for \$260, paying \$2 each for the pigs, \$4 each for the calves, and \$20 each for the cows. How many of each did he buy?

17. Divide the number 120 into three such parts that the second shall equal 3 times the first, and the third 2 times the second.

18. The sum of three numbers is 96. The second is 3 times the first, and the third is 2 times the sum of the first and second. Find the numbers.

19. Three boys, A, B, and C, picked 72 quarts of berries. A picked twice as many quarts as B, and C picked 3 times as many quarts as A. How many quarts did each pick?

20. The sum of two numbers is twice their difference, which is 34. The larger number is 3 times the smaller. What are the numbers?

21. The distance around a rectangular piece of land is 192 rods. Find the dimensions if the length is twice the width.

22. In a mixture of 63 bushels of grain, there are 4 times as many bushels of oats as rye, and one half as many bushels of corn as oats. How many bushels of each are there?

LESSON 30

$\frac{1}{2}$ of x is written $\frac{1}{2}x$, or $\frac{x}{2}$; $\frac{1}{3}$ of x is written $\frac{1}{3}x$, or $\frac{x}{3}$; $\frac{2}{3}$ of $x = \frac{2x}{3}$.

Just as $1 = \frac{2}{2}, \frac{3}{3}, \frac{4}{4}$, etc., so $x = \frac{2x}{2}, \frac{3x}{3}, \frac{4x}{4}$, etc.

1. A man and his son earned \$66. How much did each earn if the son earned one half as much as his father?

OPERATION

Let x = what the father earned,

then $\frac{x}{2}$ = what the son earned,

and $x + \frac{x}{2} = \$66$, what both earned;

$\frac{2x}{2} + \frac{x}{2} = \frac{3x}{2} = \$66$,

$\frac{x}{2} = \$22$, what the son earned,

$x = \$44$, what the father earned. } *Ans.*

2. A dealer sold a piano for $400 and thereby gained $\frac{1}{3}$ of what he paid for it. How much did he pay for it?

3. The sum of two numbers is 450, and the smaller is $\frac{2}{3}$ of the larger. What are the numbers?

4. Mary's age increased by $\frac{3}{4}$ of her age equals 21 years. How old is Mary?

5. A man lost $\frac{2}{3}$ of his money, and then had $2000 remaining. How much had he at first?

6. The sum of $\frac{1}{2}$ and $\frac{1}{3}$ of what a teacher receives in a month equals $75. How much does he receive per month?

OPERATION

Let x = what he receives,

then $\frac{x}{2}+\frac{x}{3}=\$75$,

or $\frac{3x}{6}+\frac{2x}{6}=\$75$;

$\frac{5x}{6}=\$75$,

$\frac{x}{6}=\$15$,

$x=\$90$. *Ans.*

7. The sum of $\frac{1}{3}$ and $\frac{1}{4}$ of what number equals 280?

LESSON 31

Find the values of x, y, and z in the following equations:

1.	2.	3.
$x+\frac{x}{3}=24$	$x+\frac{2x}{3}=30$	$2x-\frac{x}{3}=15$
$\frac{4x}{3}=24$	$\frac{5x}{3}=30$	$\frac{5x}{3}=15$
$\frac{x}{3}=6$	$\frac{x}{3}=6$	$\frac{x}{3}=3$
$x=18$. *Ans.*	$x=18$. *Ans.*	$x=9$. *Ans.*

4. $x+\frac{x}{4}=25$. 7. $x-\frac{2x}{3}=12$. 10. $2y-\frac{2y}{3}=28$.

5. $x+\frac{3x}{4}=42$. 8. $x-\frac{3x}{4}=11$. 11. $2x+\frac{2x}{5}=36$.

6. $x+\frac{x}{5}=36$. 9. $y-\frac{2y}{5}=27$. 12. $2z-\frac{3z}{5}=42$.

Find the value of x.

13.

$$\frac{x}{2}+\frac{x}{3}=25$$

$$\frac{3x}{6}+\frac{2x}{6}=25$$

$$\frac{5x}{6}=25$$

$$\frac{x}{6}=5$$

$x=30$. *Ans.*

14.

$$\frac{2x}{3}-\frac{x}{2}=12$$

$$\frac{4x}{6}-\frac{3x}{6}=12$$

$$\frac{x}{6}=12$$

$x=72$. *Ans.*

15.

$$\frac{4x}{5}-\frac{x}{2}=21$$

$$\frac{8x}{10}-\frac{5x}{10}=21$$

$$\frac{3x}{10}=21$$

$$\frac{x}{10}=7$$

$x=70$. *Ans.*

16. $\frac{2x}{3}+\frac{x}{2}=35$. 19. $\frac{x}{8}+\frac{x}{4}=36$. 22. $\frac{x}{10}+\frac{x}{5}=30$.

17. $\frac{x}{4}+\frac{x}{3}=21$. 20. $\frac{5x}{8}-\frac{x}{2}=9$. 23. $\frac{3x}{4}-\frac{3x}{10}=27$.

18. $\frac{x}{5}+\frac{x}{2}=42$. 21. $\frac{4x}{5}-\frac{3x}{4}=20$. 24. $\frac{2x}{3}+\frac{3x}{4}=51$.

LESSON 32

1. A certain number increased by 7 equals 29. What is the number?

We let x = the number; then $x+7$ will equal the number increased by 7. Now, since x *increased* by 7 equals 29, it is evident that x equals 29 *minus* 7, or 22.

OPERATION

Let x = the number,
then $x+7=29$,
$x=29-7$,
$x=22$. *Ans.*

2. John and William together have $45. John has $5 more than William. How much money has each.

We let x = William's money; then, since John's money is $5 more than William's, $x + \$5$ will equal John's money. Adding, we get $2x + \$5 = \$45$.

If $2x$ *increased* by $\$5 = \$45$, it is evident $2x$ will equal $45 *minus* $5, or $40.

If $2x = \$40$, $x = \$20$, William's money; and $x + \$5$, or $25 = John's money.

OPERATION

Let x = William's money.
then $x + \$5$ = John's money.
and $2x + \$5 = \$45$.
$2x = \$45 - \$5$.
$2x = \$40$.
$x = \$20$, William's money.
$x + \$5 = \$25$, John's money. } *Ans.*

3. A man bought a pair of gloves and a necktie for $2.50. He paid 50 cents less for the necktie than for the gloves. How much did he pay for each?

We let x = cost of gloves; then, since the necktie cost $.50 less than the gloves, $x - \$.50$ will equal the cost of the necktie. Adding, we get $2x - \$.50 = \$2.50$. If $2x$ *diminished* by $.50 equals $2.50, it is evident that $2x$ equals $2.50 *increased* by $.50, or $3.00.

OPERATION

Let x = cost of gloves.
then $x - \$.50$ = cost of necktie.
and $2x - \$.50 = \$2.50$.
$2x = \$2.50 + \$.50$.
$2x = \$3.00$.
$x = \$1.50$, cost of gloves.
$x - \$.50 = \$1.00$, cost of necktie. } *Ans.*

4. If Theodore's age were increased by 9 years, the sum would equal 25 years. How old is Theodore?

5. A lady bought a coat and hat for $26. She paid $6 more for the coat than for the hat. How much did she pay for each?

6. If 2 times the distance between two cities were increased by 39 miles, the sum would equal 109 miles. What is the distance?

7. Mary solved 4 problems more than her sister. They together solved 16. How many did each solve?

8. Three fourths of a certain number minus 9 equals 36. What is the number?

9. Two brothers, Elmer and Martin, earned $52 delivering papers. Martin earned $12 less than Elmer. How much did each earn?

10. A man bought a horse and sold it at a gain equal to $\frac{1}{4}$ of the cost. How much did the horse cost if he received $175 for it?

Find the value of x, y, etc., in the following equations:

11. $x+16=45$.
12. $x+24=76$.
13. $y+32=79$.
14. $x-12=27$.
15. $y-32=74$.
16. $z-36=49$.
17. $y+32=76$.
18. $\frac{3x}{4}+30=90$.
19. $\frac{2x}{3}-21=49$.
20. $\frac{4y}{5}-54=42$.
21. $2y+16=64$.
22. $3x-24=42$.
23. $2z-41=29$.
24. $2\frac{1}{2}x-12=193$.
25. $3\frac{1}{3}y+16=156$.
26. $4\frac{2}{3}x-18=94$.

LESSON 33

Multiplication is usually indicated without the use of the sign of multiplication.

Thus, $a \times b = ab$; $x \times y = xy$; $3 \times a \times b \times c = 3abc$.

When $a = 2$, $b = 3$, $c = 4$, $d = 5$, find the numerical value of the following expressions.

Thus, $4a + 3b = 8 + 9 = 17$; $3d - 2c = 15 - 8 = 7$.

1. $2a + 3c =$	4. $abc + 3d =$	7. $abc + abd - 5d =$
2. $3c - 2b =$	5. $8abc + ad =$	8. $2ad \div d =$
3. $3abc + d =$	6. $3bc \div 12 =$	9. $3abc \div 2c =$

Since the two members of an equation are equal, we may *add to*, or *subtract from*, both members equal quantities without destroying the equality. In the equation $7 + 6 = 13$, if we add 5 to each member, we have $7 + 6 + 5 = 13 + 5$.

Again, in the equation $21 = 12 + 9$, if we subtract 8 from each member, we have $21 - 8 = 12 + 9 - 8$. It will be seen that in both cases the members remain equal.

10. Find the value of x in the equation $4x - 9 = 2x + 7$. Since the known and unknown quantities appear in both members of the equation, to find the value of x we must transpose the unknown terms to the first member and the known terms to the second member. We can change any term from one member to the other by simply changing its sign.

Subtracting $2x$ from both sides of the equation, we have

$$4x - 2x - 9 = 7$$

Adding 9 to both members, we have,

$$4x - 2x = 7 + 9$$

Reducing, we have

$$2x = 16$$
$$x = 8$$

Find the value of x in the following equations:

11. $8x = 4x + 20$.	15. $12x - 4x + 8 = 32 - 16$.
12. $5x = 2x + 21$.	16. $6x - 40 + 2x = 88 - 8x$.
13. $12x - 9 = 5x + 12$.	17. $9x + 18x = 54 - 3x + 6$.
14. $9x + 12 = 60 - 3x$.	18. $8x + 8 - 6x = 32 - 4x$.

LESSON 34

1. A man bought a horse and carriage for $180. Find the cost of each, if twice what he paid for the carriage plus $15 equals what he paid for the horse.

OPERATION

Let x = cost of carriage.

then $2x + \$15$ = cost of horse.

Adding, we have $2x + x + \$15 = \$180$ = cost of both.

Transposing and collecting, we have

$$3x = \$165.$$

$$\left.\begin{aligned} x &= \$55, \text{ cost of carriage.} \\ 2x + \$15 &= \$125, \text{ cost of horse.} \end{aligned}\right\} \textit{Ans.}$$

2. Walter had twice as many marbles as Alfred. After Walter had lost 10 of his and Alfred had won 5, they together had 55. How many had each at first?

OPERATION

Let x = Alfred's number.

then $2x$ = Walter's number.

$x + 5$ = Alfred's after winning 5.

$2x - 10$ = Walter's after losing 10.

$$2x - 10 + x + 5 = 55.$$

$$3x - 5 = 55.$$

$$3x = 60.$$

$$\left.\begin{aligned} x &= 20, \text{ Alfred's number.} \\ 2x &= 40, \text{ Walter's number.} \end{aligned}\right\} \textit{Ans.}$$

3. I had 200 lb. of sugar put up in 10-pound and 5-pound packages. There were 22 more 5-pound packages than 10-pound packages. Find the number of packages of each kind.

OPERATION

Let x = number 10-pound packages.
then $x + 22$ = number 5-pound packages.
$10x$ = lb. in the 10-pound packages.
$5x + 110$ = lb. in the 5-pound packages.
$10x + 5x + 110 = 200$.
$15x = 90$.
$x = 6$, number 10-pound packages.
$x + 22 = 28$, number 5-pound packages. } *Ans.*

4. A man paid 25% of a month's wages for an overcoat. If he had \$60 remaining, how much were his monthly earnings?

5. Six times a certain number minus 10 equals 4 times the same number plus 8. What is the number?

OPERATION

Let x = the number.
then $6x - 10 = 4x + 8$.
Transposing $6x - 4x = 8 + 10$.
Reducing $2x = 18$.
$x = 9$. *Ans.*

6. Five times a certain number equals 4 times the sum obtained by increasing the same number by 2. What is the number?

7. Three brothers have together \$1690. Twice what the first has, diminished by \$220, equals what the second has, and the third has \$250 more than the second. How much money has each?

8. A and B commenced business with an equal sum of money. The first year A gained 25% of his money, and B lost 20%. They then had together \$400 more than they had at first. How much money had each at first?

LESSON 35

1. Two brothers, Willis and Robert, earn together $112 a month. How much does each earn if Willis earns 3 times as much as Robert?

2. Divide the number 350 into two such parts that the second may be 4 times the first.

3. Five times the distance between two cities diminished by $3\frac{1}{2}$ times the distance equals 90 miles. Find the distance.

4. Divide the number 80 into three parts such that the second may be 3 times the first, and the third 2 times the second.

5. Four times my age increased by 25 yr. equals 6 times my age diminished by 31 yr. What is my age?

6. A mother's age is 4 times her daughter's age, and the difference of their ages is 36 yr. What is the age of each?

7. The sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of a number equals 15 more than $\frac{2}{3}$ of the number. What is the number?

8. Martin lost 24¢, then earned $\frac{1}{2}$ as much as he lost, and then had $\frac{3}{4}$ as much as he had at first. How much had he at first?

9. William earned $\frac{2}{3}$ as much as James, and James earned $\frac{3}{4}$ as much as Henry, and they together earned $216. Find the amount earned by each.

10. The head of a fish is 3 in. long, the tail is as long as the head plus $\frac{1}{2}$ of the body, and the body is as long as the head and tail both. What is the length of the fish?

11. Divide 25 into two parts such that the greater shall be 4 more than two times the less.

REVIEW WORK

LESSON 36

1. Define interest; principal; rate; time; amount. Make a problem in interest.

2. To find the interest when the principal, rate, and time are given. Give rule for solution.

3. To find the rate when the principal, interest, and time are given. Give rule.

4. To find the principal when the rate, interest, and time are given. Give rule.

5. To find the time when the principal, rate, and interest are given. Give rule.

6. Find the interest of $650, by the exact method, from Jan. 1, 1901, to Jan. 1, 1902, at 6%. Find the interest also by the common method. Do the two methods agree?

7. Find the interest of $650 for 100 da. at 6% by the exact method. By the common method. Do the two methods agree?

8. Is it true that the two methods agree for a whole number of years? Which is greater for a fraction of a year?

9. What part of 365 da. is 5 da.? Can you show that *exact* interest for any number of days less than a year may be found by deducting $\frac{1}{73}$ of the interest found by the common method?

10. John's age is $\frac{3}{4}$ of his sister's age, and the sum of their ages equals 42 yr. Find by algebra the age of each.

11. Harry picked 5 times as many quarts of berries as Willie. Johnson picked 3 times as many quarts as Willie. They all picked 63 qt. Find, by algebra, how many quarts each picked.

12. A man's horses, sheep, and cows are worth \$3600. The cows are worth twice as much as the sheep, and the horses are worth as much as the cows and sheep together. Find, by algebra, the value of each.

LESSON 37

1. If 15 men can do a piece of work in 24 da., in how many days can 30 men do three times as much work?

2. Forty men are engaged to do a piece of work in 30 days. After 10 da., 20 men stop work. In how many days can the rest finish the work?

3. At \$18 a thousand feet, how much will it cost to board up the gable ends of a shed 36 ft. wide, the ridge being 12 ft. high?

4. I bought Scranton city bonds at 95. If they pay 5%, what is the rate of income on the investment?

5. What is the cost of insuring a cargo of wheat worth \$6400 for $\frac{3}{4}$ of its value at $1\frac{3}{4}\%$?

6. Define insurance; policy; premium.

7. A man paid \$3750 for insuring property for $\frac{3}{5}$ of its value at $2\frac{1}{2}\%$. Find the full value of the property.

8. A, B, and C hire a pasture for \$102. A puts in 3 horses, B 12 sheep, and C 13 sheep. What part should each pay if a horse eats as much as 3 sheep?

9. State the rule for finding the volume of a pyramid; for finding the entire surface.

10. Find the convex surface of a triangular pyramid each side of whose base is 16 in. and slant height 28 in.

11. I have a cylinder whose diameter is 3 in. and altitude 6 in., and I have also a cone the diameter of whose base is 3 in. and altitude 18 in. How do their diameters compare? Their altitudes? Their volumes?

12. Give rule for finding the volume of a cylinder; for finding the volume of a cone.

13. Find the volume of the greatest cylinder that can be placed in a cubical box 8 in. deep.

14. A pyramid and a cube have equal bases containing 25 sq. in. What must be the height of the pyramid so that the two figures may have equal volumes?

15. An agent charged $190 for selling a house for $9500. Find the rate of commission.

16. Find the face of a note, which when discounted at bank for 60 da., at 6%, may yield $320.

17. Find the cash value of a bill of goods amounting to $4430 at a discount of 20%, and 5% off for cash.

INVOLUTION

LESSON 38

Involution treats of the formation of powers of numbers.

A **Power** of a number is the product arising from using the number a certain number of times as a factor. The number itself is the **First Power**, or the **Root** of the other powers.

The **Second Power** or **Square** of a number is the product of the number taken twice as a factor. Thus, $4 \times 4 = 16$ = the second power, or square of 4.

The **Third Power** or **Cube** is the product of the number taken three times as a factor. Thus, $4 \times 4 \times 4 = 64$ = the third power, or cube of 4.

The **Fourth Power** is the product of the number taken four times as a factor. Thus, $4 \times 4 \times 4 \times 4 = 256$ = the fourth power of 4.

The required power of a number is indicated by a small figure called an **Exponent**. It is placed to the right of the number and a little above it. Thus, 4^2 = the second power, or the square of 4; 4^3 = the third power, or the cube of 4; 4^4 = the fourth power of 4.

1. What is the square of 2? 3? 4? 9? 12?
2. What is the second power of 6? 7? 8? 11?
3. What is the third power, or cube of 2? 3? 4? 5?
4. What is the square of $\frac{1}{2}$? $\frac{1}{4}$? $\frac{3}{4}$? $\frac{1}{3}$? $\frac{2}{3}$? $\frac{7}{8}$? .4?
5. What is the cube of $\frac{1}{2}$? $\frac{1}{3}$? $\frac{2}{3}$? $\frac{1}{4}$? $\frac{3}{4}$? .5? .7?

LESSON 39

1. Find the powers indicated: 8^2; 9^3; 13^2; 20^2; 25^2; 12^3; $(1\frac{1}{2})^2$; $.9^2$; 3.5^2; $(3\frac{2}{3})^3$; $(4\frac{3}{5})^4$.

2. What is the fifth power of 7? The fifth power of 8?

3. What is the value of $4^2 \times 5^2$? Of $4^3 \times 8^2$? Of $10^3 \times 12^2$?

4. What is the value of $(8\frac{2}{3})^2$? Of $(2\frac{1}{3})^4$? Of $(16\frac{2}{3})^2$?

5. What is the value of $(6.2)^2 \div (4.5)^2$? Of $(1\frac{1}{12})^2 \times 4^3$?

6. What is the value of $(2\frac{1}{4})^3 + (5\frac{1}{2})^2$? Of the third power of $\frac{4\frac{1}{5}}{9}$?

7. What is the value of $.006^3$? Of $(30.06\frac{1}{2})^2$?

8. If 7^3 is multiplied by 7^4, what power of 7 shall we have?

9. What power of 9 equals $9^2 \times 9^4 \times 9^3 \times 9^5$?

10. What power of $\frac{3}{4}$ is equal to $(\frac{3}{4})^2 \times (\frac{3}{4})^3 \times (\frac{3}{4})^4 \times (\frac{3}{4})^5$?

11. What power of a number is the square of the square of a number?

12. What power of a number is the square of the cube of a number?

13. What power of a number is the cube of the square of a number?

14. What is the square of $\frac{1}{2}$ of 8?

15. What is $\frac{1}{2}$ of the square of 8?

16. The square of $\frac{1}{2}$ of a number is what part of $\frac{1}{2}$ of the square of the same number?

17. The cube of $\frac{1}{3}$ of a number is what part of $\frac{1}{3}$ of the cube of the same number?

18. What number multiplied by 5 equals the square of that number?

EVOLUTION

LESSON 40

Evolution treats of finding any root of a number.

A **Root** of a number is the number itself or one of its equal factors.

The **Square Root** of a number is one of its two equal factors. Thus, the square root of 16 is 4, for $4 \times 4 = 16$.

The **Cube Root** of a number is one of its three equal factors. Thus, the cube root of 27 is 3, for $3 \times 3 \times 3 = 27$.

The **Fourth Root** of a number is one of its four equal factors; the **Fifth Root** one of its five equal factors, etc.

The **Radical Sign** ($\sqrt{}$) is the symbol of evolution. Thus, $\sqrt{25}$ indicates the square root of 25; $\sqrt[3]{27}$ indicates the cube root of 27; $\sqrt[4]{256}$ indicates the fourth root of 256.

The small figure placed in the angle of the sign is the index of the root. It indicates what root is to be found. When the sign is used without any figure added, the square root is indicated. Evolution may be indicated also by a common fraction. Thus, $9^{\frac{1}{2}}$ denotes the square root of 9; $27^{\frac{1}{3}}$ denotes the cube root of 27; $4^{\frac{3}{2}}$ denotes the square root of the third power of 4; $6^{\frac{2}{3}}$ denotes the cube root of the second power of 6.

1. What is the square root of 4? 9? 16? 25? 36? 49? 81? 100? $\frac{1}{4}$? $\frac{1}{9}$? $\frac{4}{9}$? $\frac{1}{16}$? $\frac{9}{16}$? $\frac{25}{36}$? $\frac{64}{81}$?

2. What is the cube root of 8? 27? 64? 125? $\frac{1}{8}$? $\frac{8}{27}$? $\frac{27}{64}$?

3. What is the length of each side of a square that contains 81 square inches? 144 sq. in.?

4. What is the length of one edge of a cube that contains 64 cubic inches? 125 cu. in.?

5. What is the length of each side of a square that contains 900 square rods?

SQUARE ROOT

LESSON 41

A **Perfect Square** is the product of two equal factors. A perfect square, therefore, is a number whose square root can be found exactly.

The following are examples of perfect squares:

Equal Factors		Perfect Squares	Equal Factors		Perfect Squares
1×1	=	1	10×10	=	100
2×2	=	4	99×99	=	9801
3×3	=	9	100×100	=	10000
4×4	=	16	999×999	=	998001
9×9	=	81	1000×1000	=	1000000

From the preceding examples, it may be seen that the square of a number consisting of *one* figure is composed of one or two figures; the square of a number consisting of *two* figures is composed of three or four figures; the square of a number consisting of *three* figures is composed of five or six figures, etc. Likewise, it may be shown that the square of any number is composed of twice as many figures as the number, or twice as many *minus* one. Therefore the square root of a number composed of *one* or *two* figures consists of *one* figure; the square root of a number composed of three or four figures consists of two figures; the square root of a number composed of five or six figures consists of three figures, etc.

It will be seen, therefore, that for every two figures in the square there will be one figure in the root.

Hence to ascertain the number of figures in the square root of a number, we begin at units and separate the number into periods of two figures each, and the number of periods will equal the number of figures in the root. Thus, the square root of 2025 will consist of two figures (20′25); and the square root of 63,846 will consist of three figures (6′38′46).

1. Find the square of 45.

45 equals 40 + 5, or 4 tens and 5 units

$$\begin{array}{rrrrl} & 40 & +5 & = & 45 \\ & 40 & +5 & = & 45 \\ \hline (t \times u) + u^2 = & 5 \times 40 & +5^2 & = & 225 \\ t^2 + (t \times u) = & 40^2 + & 5 \times 40 & = & 1800 \\ \hline t^2 + 2(t \times u) + u^2 = & 40^2 + 2(5 \times 40) & +5^2 & = & 2025 \quad \textit{Ans.} \end{array}$$

Commencing at units, 5 times $5 = 5^2$ = the square of the units. 5 times $40 = 5 \times 40$ = the product of the tens and units. 40 times $5 = 5 \times 40$ = the product of the tens and units. 40 times $40 = 40^2$ = the square of the tens. Adding, we have $40^2 + 2(5 \times 40) + 5^2$. Hence it will be seen that the square of 45, or of any number consisting of tens and units, equals the square of the tens, plus twice the product of tens and units, plus the square of the units.

2. Find the square root of 2025.

ANALYTIC PROCESS

	20′25	40 = 4 tens	
Square of the tens = 40^2 =	1600	5 = 5 units	
Trial divisor = twice the tens = 2×40 =	80	425	45 = *Ans.*
Twice the product of the tens and units $= 2(5 \times 40)$ =	400		
Square of the units = 5^2 =	25	425	

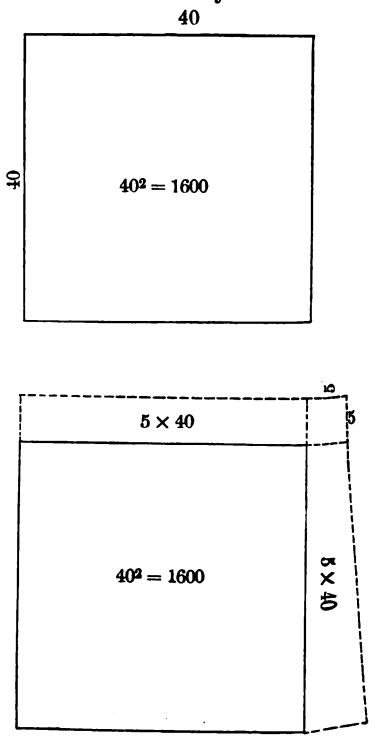
We find (p. 66) that the root must consist of two figures, tens and units. The greatest number of tens whose square is contained in

2025 is 4 tens, or 40, whose square is 1600. This, subtracted from 2025, leaves 425, which, according to Art. 628, must equal twice the product of the tens and units, plus the square of the units. To find the units we must divide 425 by twice the tens, or 80. Dividing 425 by 80, we find the units to be 5. Then two times the product of the tens and units $= 2(5 \times 40) = 400$. Adding to this the square of the units, or 25, we have 425, which, subtracted, leaves no remainder.

Illustration by Areas

If 2025 is a perfect square, it can be illustrated by the area of a square the side of which is the square root of 2025.

The greatest number of tens whose square is contained in 2025 is 4 tens, or 40. If we let Fig. 1 represent an approximate square whose sides are 40 units, its area will exhaust 40×40, or 1600 square units of the area. Subtracting 1600 from 2025, we have 425 square units of surface to be added to two sides of the approximate square, 1600, to make it the required square, 2025. This requires two rectangular pieces each 40 units long, and a square corner piece with sides equal to the width of the rectangles. The entire length of the two rectangular side pieces equals $40 + 40$, or 80 units. Dividing 425, which is the approximate area of the two side pieces, by 80, we get their width, which is 5. The areas of the two rectangular sides, and of the corner piece, are



$5 \times 40 = 200$ = one side piece.
$5 \times 40 = 200$ = the other side piece.
$5 \times 5 = 25$ = the corner piece.
425, total.

This exhausts the area, and the side of the required square is $40 + 5$, or 45. Therefore the square root of 2025 is 45.

NOTE. — In practice we shorten the process as in the following example.

3. Find the square root of 104,976.

Pointing off into periods of two figures each, we find there are three periods, and therefore three figures in the root.

OPERATION

10′49′76(324 *Ans.*
$3^2 = 9$
62)149
124
644)2576
2576

The greatest number whose square is contained in the left-hand period is 3. We place the 3 in the root, and subtract its square from 10, and bring down the next period, 49, making the dividend 149. We double the first figure of the root, 3, making 6 for a trial divisor, which we write at the left of 149. Six is contained in 14 *twice*. We write the 2 in the root and in the divisor also, making the divisor 62. We multiply 62 by the second figure of the root and obtain the product 124. Subtracting this from 149 and bringing down the next period, we have 2576 for a dividend. We next double the root already found, 32, thus making 64, which we write at the left of 2576, for a trial divisor. It will be seen that 64 is contained in 257 *four* times. We write the 4 in the root, and also in the divisor, thus increasing the divisor to 644. We next multiply 644 by the last figure of the root, 4, and obtain the product 2576. As nothing remains, the square root of 104,976 equals 324.

4. Find the square root of 249,001.
5. Find the square root of 316,969.
6. Find the square root of 349,281.
7. Find the square root of 484,416.
8. Find the square root of 574,564.

LESSON 42

The left-hand period may consist of but one figure. When the trial divisor is larger than the dividend, exclusive of the right-hand figure, the corresponding figure of the root will be a cipher. When this is the case we annex a cipher to the trial divisor and bring down the next period.

1. Find the square root of 42,025.

It will be seen that the left-hand period contains but one figure. The greatest number whose square is contained in 4 is 2. We write the 2 in the root and its square under the 4 and bring down the next period, 20. We next double the first figure of the root, 2, making 4 for a trial divisor, which is written at the left of 20. When the trial divisor is larger than the dividend, exclusive of the right-hand figure, we annex a cipher to both the root and the trial divisor, bring down the next period, and then proceed as in Ex. 3, page 69.

OPERATION

4′20′25(205 *Ans.*
$2^2 = 4$
405)2025
2025

To extract the square root of a decimal we point off the decimal *toward the right* into periods of two figures each, instead of toward the left, as in whole numbers, and extract the root as in the preceding examples. When the last period contains but *one* figure, we annex a cipher to complete the period.

2. Find the square root of .119025.

OPERATION

.11′90′25(.345 *Ans.*
$3^2 = 9$
64)290
256
685)3425
3425

When there is a remainder after all the periods have been brought down, we annex periods of ciphers and continue the operation to as many decimal places as may be desired.

3. Find the square root of .59049.

OPERATION

.59′04′9(.76843 + *Ans.*

$7^2 = 49$

146)1004

876

1528)12890

12224

15364)66600

61456

153683)514400

461049

53351 Rem.

To extract the square root of a whole number and a decimal, we point off the whole number toward the left and the decimal toward the right.

4. Find the square root of 522.064.

OPERATION

5′22.06′4(22.848 + *Ans.*

$2^2 = 4$

42)122

84

448)3806

3584

4564)22240

18256

45688)398400

365504

32896 Rem.

LESSON 43

To extract the square root of a common fraction, when both terms are perfect squares, we extract the square root of the numerator and denominator separately. Thus, $\sqrt{\frac{9}{16}} = \frac{3}{4}$, for $\sqrt{9} = 3$ and $\sqrt{16} = 4$.

When both terms of a common fraction are not perfect squares, we reduce the fraction to a decimal and extract the root of the decimal. Thus, $\sqrt{\frac{7}{8}} = \sqrt{.875} = .935+$. Sometimes the fraction may be reduced so that both terms become perfect squares. Thus, $\sqrt{\frac{12}{27}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$; $\sqrt{\frac{18}{32}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$; $\sqrt{21\frac{7}{9}} = \sqrt{\frac{196}{9}} = \frac{14}{3} = 4\frac{2}{3}$.

To extract the square root of a mixed number it is usually best to reduce the fraction to a decimal. Thus,

$$\sqrt{7\frac{2}{3}} = \sqrt{7.6666+} = 2.76+.$$

When the denominator is a perfect square, we may extract the square root of the numerator and denominator separately. Thus, $\sqrt{\frac{11}{64}} = \frac{\sqrt{11}}{\sqrt{64}} = \frac{3.316+}{8} = .414+$.

Find the square root of:

1. 196.
2. 361.
3. 576.
4. 2116.
5. 6241.
6. 46,656.
7. 94,249.
8. 189,225.
9. 527,076.
10. 649,636.
11. .0576.
12. .1225.
13. .0064.
14. .00261.
15. .170569.
16. 53.29.
17. 1162.25.
18. 7397.7201.
19. $\frac{64}{81}$.
20. $\frac{196}{225}$.
21. $\frac{324}{361}$.
22. $\frac{441}{625}$.
23. $\frac{147}{192} = \frac{49}{64}$.
24. $\frac{3}{7}$.
25. $7\frac{1}{2}$.
26. $9\frac{4}{7}$.
27. $4\frac{11}{12}$.
28. $76\frac{9}{16} = \frac{1225}{16}$.
29. $2\frac{1}{4} \times 6\frac{1}{2}$.
30. $\frac{576}{676} = \frac{144}{169}$.
31. $12.6\frac{7}{8}$.
32. $25.0\frac{3}{4}$.
33. $.000\frac{4}{9}$.

LESSON 44

1. If it requires 144 yd. of matting 1 yd. wide to cover the floor of a square room, what is the length of each side of the room in feet?

NOTE. — Since the area of a square is the product of two of its sides, the length of one side is obtained by extracting the square root of its area.

2. I own a square piece of land which contains just 40 A. What is the length of each side in rods?

3. What is the side of a square whose area is $272\frac{1}{4}$ sq. ft.?

4. One half the square of a certain number is 275,282. What is the number?

5. How much will it cost, at \$.80 a rod, to fence a square lot containing 245,025 sq. ft.?

6. How many yards long is each side of a square reservoir which contains 10 acres?

7. How large a square table can be made from two boards, each 12 ft. long, and 18 in. wide?

8. How much less will it cost to inclose a square piece of land containing 40 A., than one containing the same area in the form of a rectangle 90 rd. long, the price per rod being \$.81 in each case?

9. A rectangular piece of land is 4 times as long as it is wide, and contains just 10 acres. Find its dimensions.

10. A gentleman has a garden which contains 3600 sq. ft. Its length is to its width as 9 to 4. Find its dimensions.

CUBE ROOT

LESSON 45

1. What is the cube root of a number?

2. Review examples 2 and 4, Lesson 40.

A **Perfect Cube** is the product of three equal factors. It is, therefore, a number whose cube root can be found exactly. The following shows how perfect cubes are produced.

Equal Factors	Perf. Cubes	Equal Factors	Perf. Cubes
$1 \times 1 \times 1 =$	1	$10 \times 10 \times 10 =$	1000
$2 \times 2 \times 2 =$	8	$46 \times 46 \times 46 =$	97,336
$3 \times 3 \times 3 =$	27	$99 \times 99 \times 99 =$	970,299
$4 \times 4 \times 4 =$	64	$100 \times 100 \times 100 =$	1,000,000
$5 \times 5 \times 5 =$	125	$444 \times 444 \times 444 =$	87,528,384
$9 \times 9 \times 9 =$	729	$999 \times 999 \times 999 =$	997,002,999

$$1000 \times 1000 \times 1000 = 1{,}000{,}000{,}000$$

From the preceding examples of perfect cubes it may be seen that the cube of a number consisting of *one* figure is composed of *one*, *two*, or *three* figures; the cube of a number consisting of *two* figures is composed of *four*, *five*, or *six* figures; the cube of a number consisting of *three* figures is composed of *seven*, *eight*, or *nine* figures; the cube of a number consisting of *four* figures is composed of *ten*, *eleven*, or *twelve* figures, etc. Likewise it may be shown that the cube of any number is composed of three times as many figures as the number itself, or three times as many, minus *one* or minus *two*.

Therefore the cube root of a number composed of *one*, *two*, or *three* figures consists of but *one* figure; the cube root of a number composed of *four*, *five*, or *six* figures consists of *two* figures; the cube root of a number composed of *seven*, *eight*, or *nine* figures consists of *three* figures; the cube root of a number composed of *ten*, *eleven*, or *twelve* figures consists of *four* figures, etc.

Therefore for every *three* figures in the cube there will be *one* figure in the root.

Hence, to ascertain the number of figures in the cube root of a number, we separate the number into periods of three figures each, beginning at units and pointing off toward the left. The number of periods will denote the number of figures in the root.

LESSON 46

1. Find the cube of 45.

OPERATION I

$45 = 40 + 5$, or 4 tens and 5 units.

$45^2 = (40+5)^2 = 40^2 + 2(40 \times 5) + 5^2 = 2025$ (p. 67)

$40 + 5 = 45$

$(40^2 \times 5) + 2(40 \times 5^2) + 5^3 = 10125$

$40^3 + 2(40^2 \times 5) + (40 \times 5^2) = 81000$

$40^3 + 3(40^2 \times 5) + 3(40 \times 5^2) + 5^3 = 91125$ *Ans.*

OPERATION II

$40 + 5 = 45$

$40 + 5 = 45$

$200 + 25 = 225$

$1600 + 200 = 1800$

$1600 + 400 + 25 = 2025$

$40 + 5 = 45$

$8000 + 2000 + 125 = 10125$

$64000 + 16000 + 1000 = 81000$

$64000 + 24000 + 3000 + 125 = 91125$ *Ans.*

A careful inspection of the two preceding operations will show us that the cube of a number consisting of tens and units embraces the following four parts :

(*a*) The cube of the tens, $40^3 = 64{,}000$.

(*b*) Three times the product of the square of the tens by the units, $3(40^2 \times 5) = 24{,}000$.

(*c*) Three times the product of the tens by the square of the units, $3(40 \times 5^2) = 3000$.

(*d*) The cube of the units, $5^3 = 125$.

LESSON 47

1. Find the cube root of 91,125.

ANALYTIC PROCESS

		91'125	40 = 4 tens
Cube of the tens	= 40^3 = 64000		5 = 5 units
Trial divisor = 3 times the square of the		27125	45 *Ans.*
tens	= 3×40^2 = 4800		
3 times the square of the tens times the			
units	= $3(40^2 \times 5)$ = 24000		
3 times the tens times the square of the			
units	= $3(40 \times 5^2)$ = 3000		
Cube of the units	= 5^3 = 125	27125	

According to Lesson 45, the number, 91,125, has a cube root of two figures, viz. *tens and units*. This number, 91,125, according to Lesson 46, must equal the cube of the tens plus 3 times the square of the tens multiplied by the units, plus 3 times the tens multiplied by the square of the units, plus the cube of the units.

The greatest number of tens whose cube is contained in 91,125 is 4 tens, or 40, whose cube is 64,000. This subtracted from 91,125 leaves 27,125, which, according to Lesson 46, must equal 3 times the square of the tens multiplied by the units, plus 3 times the tens multiplied by the square of the units, plus the cube of the units.

We now have the *tens* of the root. We must next find the units.

As the remainder, 27,125, must contain the products just mentioned, of each of which the units form one factor, we may find the other factor by dividing by the known factor, viz. *three times the square of the tens*, or 4800. This is called the trial divisor. By way of trial, we divide 27,125 by 4800, and obtain the quotient, 5, which we write as the trial units' figure of the root.

Completing the required products we find they amount to 27,125. Hence 5 is the units' figure of the root. If the products, when completed, amount to more than the dividend, a smaller quotient figure must be taken.

In practice, we usually omit the ciphers, and delay multiplying by the units until we have found the sum of the other factors. We then multiply them all at once, as in the following operation:

OPERATION

$$\begin{array}{rl} & 91'125(45\ \textit{Ans.} \\ 4^3 = & 64 \\ \text{Trial divisor} = 3 \times 40^2 = & 4800 \mid 27125 \\ 3 \times 40 \times 5 = & 600 \\ 5^2 = & 25 \\ \text{Complete divisor} = & 5425 \mid 27125 \end{array}$$

ILLUSTRATION BY SOLIDS

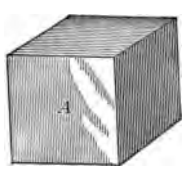


FIG. 1

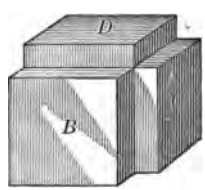


FIG. 2

If 91,125 is a perfect cube, it can be illustrated by a cubical block the length of whose edge is the cube root of 91,125.

The greatest number of tens whose cube is contained in 91,125 is 4 tens, or 40. Now, if we let *A*, Fig. 1, represent an approximate

cube, the length of whose edge is 40 units, it will exhaust $40 \times 40 \times 40$, or 64,000 cubic units of the volume. Subtracting 64,000 from 91,125, we have left 27,125 cubic units to be added to the sides of the approximate cube, 64,000, to make it a cube equal to 91,125 cubic units.

To do this we require:

1. Three side-slabs, each 40 units square (*B*, *C*, *D*, Fig. 2).

2. Three corner pieces (*E*, *F*, *G*, Fig. 3), each 40 units long, with a width and thickness equal to the thickness of the side-slabs.

3. A corner cube (*H*, Fig. 4) to fill the vacancy left by the corner pieces (*E*, *F*, *G*, Fig. 3).

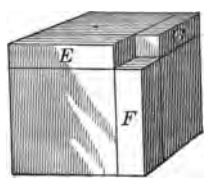


FIG. 3

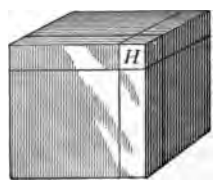


FIG. 4

Since the remainder, 27,125, consists chiefly of the three square side-slabs, we may find, nearly, their thickness by dividing 27,125 by the area of one side of the three slab-additions, which equals $40 \times 40 \times 3$, or 4800. Dividing 27,125 by 4800, we get a quotient of 5, which we find to be the exact thickness. Therefore the volume of the several additions to the approximate cube is as follows:

Three side-slabs, $B + C + D$, Fig. 2	$= 40 \times 40 \times 5 \times 3 =$	24,000
Three corner pieces, $E + F + G$, Fig. 3	$= 40 \times 5 \times 5 \times 3 =$	3000
The corner cube *H*, Fig. 4	$= 5 \times 5 \times 5 =$	125
	Total =	27,125

This exhausts the volume, and the edge of the required cube is $40 + 5$, or 45 units. Therefore, the cube root of 91,125 is 45.

LESSON 48

1. $\sqrt[3]{150,568,768} = ?$

OPERATION

150′568′768 | 532 *Ans.*

	$5^3 =$		125
Partial, or trial divisor . .	$50^2 \times 3 =$	7500	25568
	$50 \times 3 \times 3 =$	450	
	$3^2 =$	9	
First complete divisor		7959	23877
Partial, or trial divisor . .	$530^2 \times 3 =$	842700	1691768
	$530 \times 2 \times 3 =$	3180	
	$2^2 =$	4	
Second complete divisor		845884	1691768

We first separate the number into periods of three figures each, beginning at units and counting to the left. The greatest number whose cube is contained in 150 is 5. We write the 5 in the root, and subtract its cube, 125, from the first period 150, which leaves a difference of 25, to which we annex the next period, thus forming the dividend 25,568.

Annexing a cipher to the first figure of the root, squaring it, and multiplying by 3, we obtain 7500 as a partial or trial divisor ($50^2 \times 3 = 7500$). Placing 7500 to the left of 25,568 and dividing, we get the quotient 3, which forms the second figure of the root. In order to complete the divisor, we annex a cipher to the first figure of the root, multiply it by the second figure of the root, and the product by 3, and obtain 450 ($50 \times 3 \times 3 = 450$) as the second part of the first complete divisor. We next square 3, the second figure of the root, write it under 450, and add these partial divisors together, which gives 7959 for the first complete divisor. Multiplying the complete divisor by the second figure of the root, 3, and subtracting the product, 23,877, from 25,568, we have a difference of 1691, to which we annex the next period, and obtain 1,691,768 for a dividend.

We now annex a cipher to the root already found, 53, square it, multiply by 3, and obtain 842,700 as a partial, or trial divisor ($530^2 \times 3 = 842,700$). Placing the trial divisor to the left of 1,691,768 and dividing, we get the quotient 2, which forms the third figure of

the root. Annexing a cipher to the first two figures of the root, multiplying it by 2, the third figure of the root, and this product by 3, we obtain 3180 ($530 \times 2 \times 3 = 3180$), which we write under 842,700, as the second part of the second complete divisor. We next square 2, the third figure of the root, write it under 3180, and add these partial divisors together, and get for the second complete divisor 845,884, which we multiply by 2, the last figure of the root; we subtract the product from 1,691,768. As there is no remainder, the cube root of 150,568,768 is 532.

The left-hand period may contain only one or two figures.

2.

$5'832 \mid 18$ *Ans.*

$1^3 =$	1	
$10^2 \times 3 =$	300	4832
$10 \times 8 \times 3 =$	240	
$8^2 =$	64	
	604	4832

3.

$97'336 \mid 46$ *Ans.*

$4^3 =$	64	
$40^2 \times 3 =$	4800	33336
$40 \times 6 \times 3 =$	720	
$6^2 =$	36	
	5556	33336

When the dividend will not contain the trial divisor, we write a cipher in the root, annex two ciphers to the trial divisor, bring down the next period, and proceed to complete the divisor as before.

If the number is not a perfect cube, there will be a remainder after all the periods have been brought down. When this is the case we annex periods of ciphers, and continue the operation as far as desirable. For every period of three ciphers annexed there will be a decimal figure in the root.

To find the cube root of a common fraction, we extract the cube root of both numerator and denominator separately, when they are perfect cubes, but when they are not,

it is usually best to reduce the fraction to a decimal and extract its root.

To find the cube root of a decimal, we separate the decimals into periods of three figures each, beginning at the decimal point and counting to the right.

LESSON 49

Find the cube root of the following:

1. 2744.	**6.** 223648543.	**11.** $26\frac{1}{2}$.
2. 15625.	**7.** 350402625.	**12.** .216.
3. 175616.	**8.** 259.694072.	**13.** .000343.
4. 1860867.	**9.** 12.812904.	**14.** $\frac{27}{343}$.
5. 37933056.	**10.** 16503.467336.	**15.** $\frac{512}{2197}$.

16. Find the cube root of .1 to four decimal places.

17. Find the cube root of 6 to five decimal places.

18. Find the value of $\sqrt[3]{\frac{5}{8}}$; $\sqrt[3]{18\frac{26}{27}}$; $\sqrt[3]{4699\frac{27}{64}}$; $\sqrt[3]{\frac{8}{9}}$; $\sqrt[3]{\frac{4}{27}}$.

19. Find the length of the edge of a cube whose volume is 13,824 cu. in.

20. Find the number of square feet in one face of a cube containing $91\frac{1}{8}$ cu. ft.

21. Find the depth of a cubical bin that contains just 80 bu.

22. Find the number of square feet of zinc that will be required to line a cubical tank whose contents are $421.87\frac{1}{2}$ cu. ft.

REVIEW WORK

LESSON 50

1. How many figures are there in the square root of a number expressed by one figure? By two figures? By three figures? By four figures?

2. Define involution; evolution; a perfect square.

3. What is the symbol of evolution? What is an exponent?

4. Find the true discount and present worth of $944 due in 2 yr. 6 mo., at 6%.

5. Measure your schoolroom, and calculate:

(*a*) How many square feet in the floor.

(*b*) In the side walls, deducting door and windows.

(*c*) In the ceiling.

(*d*) How much it would cost to plaster the walls and ceiling at 27¢ a square yard.

(*e*) How much it would cost to cover the floor with Brussels carpet at $1.35 a yard, the strips of carpet to be laid lengthwise of the room.

6. How many double rolls of paper will be needed for the walls and ceiling of a room 36 ft. long, 20 ft. wide, and 12 ft. high, deducting 130 sq. ft. for windows, doors, and baseboards?

7. What is the circumference of the largest circle that can be inscribed within a 2-foot square?

8. Find the difference between the area of the circle and that of the square.

9. How many half-inch cubes can be cut from a half-inch board 4 ft. long, and 6 in. wide, not considering waste in cutting?

10. Change $\frac{10}{14}$, $\frac{16}{24}$, $\frac{4}{6}$, and $\frac{30}{35}$ to 21 sts.

11. I bought a turkey which weighed $9\frac{1}{2}$ lb. for $\$1.04\frac{1}{2}$. At the same rate find the cost of one that weighs $12\frac{3}{4}$ lb.

LESSON 51

1. How many per cent does a boy receive if he spells correctly 57 words of the 60 pronounced by his teacher?

2. By selling a quantity of goods for $\$56$, I gained $\frac{3}{4}$ of the cost. How much did the goods cost me?

3. What must be paid for 6% stock in order to realize 10% on the investment?

4. A, B, and C engaged in business. A put in $\$5000$, B $\$6000$, and C $\frac{3}{5}$ of the entire capital. They lost $\$9240$. Find the loss of each.

5. A commercial traveler sold goods in 1901 to the amount of $\$31896$, which was 20% more than he sold in 1900. Find the amount of his sales in 1900.

6. Find the area of a walk 3 ft. wide running round the outside of a circular flower bed whose diameter is 80 ft.

7. How many cubic inches must be cut away from a block of wood 4 in. square and 12 in. high to leave a cylinder having a diameter of 4 in. and altitude 12 in.?

8. Divide $\$2079$ among A, B, and C, so that their shares shall be in the ratio of $\frac{7}{12}$, $\frac{5}{9}$, and $\frac{11}{18}$.

9. A owns $32\frac{3}{5}$ acres of land, B $45\frac{1}{2}$ acres, and C owns $\frac{1}{2}$ as many acres as A and B together. How many acres does C own, and how many acres have they all?

10. How many miles equal 2° 18′ near the equator?

LESSON 52

1. A furnished \$1700 for 9 mo., B, \$1820 for 10 mo., C, \$1500 for 11 mo. They gained \$2500. Find the share of each.

2. Divide 80 into two parts such that $\frac{1}{5}$ of the first part exceeds $\frac{1}{10}$ of the second part by 9.

3. Define promissory note; bank; bank discount; proceeds; payee; maker or drawer of note.

4. Compare the volume of two cubes, the edge of one being 2 in., and of the other 4 in.

5. The width of a rectangular piece of land that contains 120 sq. rd. is 120 ft. What is the length?

6. How long is a square piece of land containing 180 sq. rd.?

7. If a train goes 200 mi. in 4 hr. 50 min., how long will it be in going 150 mi.?

8. A rectangle is 243 ft. long, 108 ft. wide. Find the side of a square that contains the same area.

9. Find the width of a 2-inch plank 16 ft. long that contains 40 board feet.

10. Bronze is composed of copper and tin in the proportion of $4\frac{1}{2}$ parts of copper to 1 part of tin. How much tin must be used with 90 lb. of copper to make bronze?

11. Brass is an alloy of copper and zinc, usually containing about $\frac{1}{3}$ of its weight of zinc. How much copper must be used with $166\frac{2}{3}$ lb. of zinc to make brass?

LESSON 53

1. How much must I invest in stocks at 85, yielding a semiannual dividend of $4\frac{1}{2}\%$, to produce an annual income of $4675?

2. At what price must a merchant mark a hat costing $2 so that he can discount the price 20% and still make 10%?

3. In 570 oz. of gold coin there are 513 oz. of gold and 28.5 oz. each of silver and copper. What per cent of the whole is each?

4. Find the difference in time between Boston and a place 105° 40′ 15″ west of it. Which has the earlier time?

5. Find the depth of a cylindrical tank that will hold just 6 barrels of water, if it is 6 ft. in circumference.

6. A certain number plus $66\frac{2}{3}\%$ of itself plus 80% of itself equals 296. What is the number?

7. Divide $60 between two men, giving the second $3 less than $\frac{1}{2}$ of what the first receives.

8. How many acres are there in a rectangular field which measures $24\frac{1}{2}$ ch. on one side and 32 ch. 25 li. on the other?

9. How many acres are there in a triangular field, one side of which measures 100 rd., if the perpendicular distance from this side to the opposite side is $18\frac{1}{2}$ chains?

10. Five sixths of a number is 10 more than $\frac{1}{2}$ of the number. Find by algebra the number.

11. A boy spent $37\frac{1}{2}\%$ of his money, and then earned 80% as much as he had left. If he then had $15 more than he had at first, how much had he at first?

SURFACES

LESSON 54

1. Define triangle; right-angled triangle, or right triangle; equilateral triangle; isosceles triangle; scalene triangle.

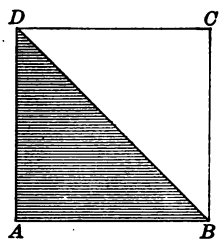


FIG. 1

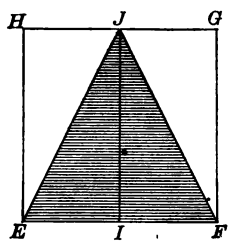


FIG. 2

A little inspection of Figs. 1 and 2 will show us that the triangle *ABD* is just one half of the rectangle *ABCD*, and the triangle *EFJ* is just one half of the rectangle *EFGH*.

The area of a triangle equals one half of a rectangle having the same base and altitude. Hence,

To find the area of a triangle, take one half of the product of its base and altitude.

2. What is the area of a triangle whose base is 24 ft. and altitude 14 ft.?

OPERATION

$(24 \times 14) \div 2 = 168$ sq. ft. *Ans.*

Either the base or altitude of a triangle is found by dividing the area by the given dimension, and multiplying the quotient by two. Explain.

3. The area of a triangle is 96 sq. ft., and the base is 16 ft. What is the altitude?

OPERATION

$(96 \div 16) \times 2 = 12$ ft., altitude. *Ans.*

4. The area of a triangle is 96 sq. ft., and its altitude 12 ft. Find the base.

OPERATION

$(96 \div 12) = 16$ ft., base. *Ans.*

To find the area of a triangle when its three sides are given, from half the sum of the three sides subtract each side separately. Find the product of the half sum and the three remainders. The square root of the product will be the area of the triangle.

5. Find the area of a triangle with sides 6, 8, and 10 ft.

OPERATION

$(6 + 8 + 10) \div 2 = 12$, half the sum of the three sides.

$12 - 6 = 6$
$12 - 8 = 4$ } = each side subtracted separately.
$12 - 10 = 2$

$\sqrt{12 \times 6 \times 4 \times 2} = 24$ sq. ft. *Ans.*

6. Find the area of a triangle whose base is 36 rd. and altitude 28 rd.

7. Find the area of a triangle whose base is 80 yd. and altitude 63 ft.

8. Find the area of a triangle whose base is 24.5 ch. and altitude 16.8 ch.

9. The area of a triangular piece of land is 15 A. The altitude is 60 rd. Find the base.

10. A triangular piece of land contains 1980 sq. yd. The base is 72 yd. Find the altitude.

11. Find the area of a triangle whose sides are respectively 15, 20, and 25 ft.

12. Find the value, at $85 per acre, of a triangular piece of land, the sides being respectively 30, 40, and 50 ch.

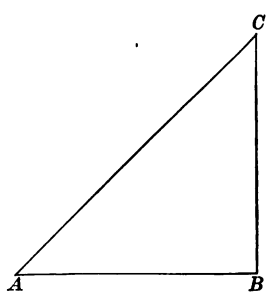
13. Find the area of an isosceles triangle whose base is 50 rd., and each of the equal sides 40 rd.

14. How much will it cost, at 5¢ a square foot, to paint the two gables of a house, if the width of the house is 32 ft., and the length of the rafters on each side is 20 ft.?

15. Which is greater and how much, a triangle whose sides are respectively 30, 40, and 50 ft., or one whose base is 60 ft. and altitude 20 ft.?

LESSON 55

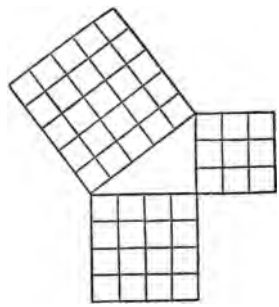
In the right-angled triangle ABC, the side AC, or the side opposite the right angle, is called the **Hypotenuse**, the side AB the **Base**, and the side BC the **Perpendicular**.



It is an established principle of geometry *that the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the base and the perpendicular.*

The diagram on p. 89 illustrates the preceding principle. It will readily be seen that the number of small

squares in the square of the hypotenuse equals the sum of the small squares in the base and the perpendicular. Hence,



The hypotenuse equals the square root of the sum of the squares of the base and perpendicular; the base equals the square root of the difference of the squares of the hypotenuse and the perpendicular; and the perpendicular equals the square root of the difference of the squares of the hypotenuse and the base.

1. The base of a right-angled triangle is 32, and the perpendicular 60. Find the hypotenuse.

STATEMENT

$$\sqrt{B^2 + P^2} = \text{answer}$$

2. The perpendicular of a right-angled triangle is 60, and the hypotenuse 65. Find the base.

STATEMENT

$$\sqrt{H^2 - P^2} = \text{answer}$$

3. The hypotenuse is 85, the base 51. Find the perpendicular.

STATEMENT

$$\sqrt{H^2 - B^2} = \text{answer}$$

4. Two men travel from the same place. One travels directly east at the rate of 16 mi. a day, and the other goes due north at the rate of 12 mi. a day. How far apart will they be at the end of 6 days?

5. The rafters of a house are 17 ft. long, and the height of the gable is 8 ft. What is the width of the house?

6. A rope 190 ft. long will reach from the top of a pole to a point on the opposite bank of a river whose width is 152 ft. What is the height of the pole?

7. How far from the bottom of a building 44 ft. high must the foot of a ladder 55 ft. long be placed so that the top of the ladder will just reach the top of the building?

8. I own a lot 200 ft. by 184 ft. What is the distance through it from opposite corners?

9. A flag pole 80 ft. high stands at the edge of a street. A rope 96 ft. long fastened at the top of the pole reaches to the middle of the street. Find the width of the street.

10. Find the distance from a lower corner to the opposite upper corner of a room 32 ft long, 24 ft. wide, and 18 ft. high.

LESSON 56

1. Define rhomboid; rhombus; trapezoid; trapezium.

2. Show by a drawing of any convenient size that the area of a rhomboid equals the product of the base and altitude.

3. Show by a drawing that the area of a rhombus equals the product of the base and altitude.

4. Show by a drawing that the area of a trapezoid equals the product of its altitude and one half of the sum of its bases.

5. Show by a drawing how to find the area of a trapezium.

6. What is the area of a piece of land in the form of a parallelogram, whose length is 30 rd. and altitude 25 rd.?

7. The base of a rhombus is 30 yd. and the altitude 70 ft. Find its area.

8. The base of a rhomboid is 30 ch. and the altitude 25 rd. What is its area?

9. Find the area of a trapezoid, one side of which is 12 ft., the other 8 ft., and the altitude 4 ft.

10. The diagonal of a trapezium is 60 ft., and the altitudes of the triangles into which the trapezium is divided are 40 ft. and 30 ft. respectively. Find the area of the trapezium.

11. How many acres are there in an irregular-shaped piece of land whose sides are 15, 20, 30, and 35 rd. respectively, and the diagonal 25 rd.?

12. The diagonal of a square is 72 ft. Find its area.

NOTE. — The area of a square equals one half of the square of its diagonal.

13. The diagonals of a rhombus are 60 ft. and 32 ft. Find the area.

14. The diagonal of a rectangle is 45 yd. What is its area, its length being to its width as 4 to 3?

LESSON 57

1. Define circle; circumference; diameter; radius; arc.

2. What is the ratio of the circumference of a circle to its diameter?

3. Illustrate by means of a drawing that the area of a circle may be found by multiplying the circumference by one fourth of the diameter.

4. (*a*) Diameter × 3.1416 = what?
(*b*) Circumference ÷ 3.1416 = what?
(*c*) Circumference × $\frac{1}{4}$ diameter = what?
(*d*) Area ÷ circumference = what?
(*e*) Area ÷ $\frac{1}{4}$ diameter = what?

5. Can you formulate a rule for finding the diameter of a circle when the area and circumference are given?

6. Can you formulate a rule for finding the circumference of a circle when the area and diameter are given?

7. Find the circumference of a circle whose diameter is 30 rd.

8. Find the diameter of a circle whose circumference is 9.4248 rd.

9. The area of a circle is 706.86 sq. rd. and the circumference is 94.248 rd. Find the diameter.

10. The area of a circle is 706.86 sq. rd. and the diameter is 30 rd. Find the circumference.

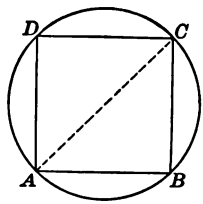
11. The base of a rhomboid is 20 ch. and the altitude 25 rd. What is its area?

12. The sides of an equilateral triangle are each 16 rd. What is the area?

LESSON 58

To find the side of an inscribed square when the circumference is given.

From the accompanying figure the following facts are apparent:



a. The diameter AC of the circle forms the diagonal of the square $ABCD$.

b. The diagonal forms the hypotenuse of the right-angled triangle ABC.

c. The square of the diagonal equals the sum of the squares of the sides AB and BC.

d. The side AB equals the side BC.

e. The square root of one half of the square of the diagonal of a square equals one side of the square. Hence,

$$AC^2 = AB^2 + BC^2 = 2\,AB^2$$

$$\frac{AC^2}{2} = AB^2 \text{ and } \sqrt{\frac{AC^2}{2}} = AB$$

1. Find the side of the largest square that can be drawn within a circle whose diameter is 20 ft.

2. Find the side of the largest square that can be drawn within a circle whose diameter is 32 yd.

3. Find the area of the largest square that can be drawn within a circle whose radius is 4 ft.

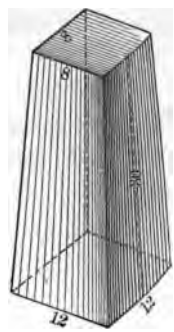
4. The area of an inscribed square is 25 sq. ft. Find the diameter of the circle.

5. Find the entire surface of an equilateral triangular pyramid, the perimeter of whose base is 18 ft., and whose slant height is 24 ft.

6. What was the length of a pole, standing 21 ft. from a building, the top of which, in falling, struck the building 28 ft. from the ground?

LESSON 59

The **Frustum** of a pyramid or cone is the part that remains after cutting off the top by a plane parallel to the base.



FRUSTUM OF A SQUARE PYRAMID

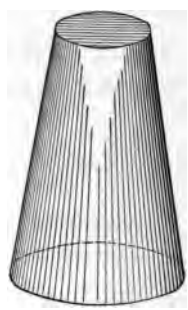
1. How many bases has a frustum? Is the upper base an exact reduced copy of the lower base?

NOTE. — The bases of a frustum of a pyramid may have any number of edges and any shape.

2. The lateral faces of a frustum of a pyramid are always what plane figures?

3. How many trapezoids form the convex or lateral surface of the above square pyramid?

4. State your rule for finding the area of a trapezoid.



FRUSTUM OF A CONE

5. If the figures represent feet, what is the area of one face of the above square frustum of a pyramid? Of the four faces?

6. What is the area of the lower base? Of the upper base?

7. What is the entire surface of the frustum?

8. Is it true that the convex surface of a frustum of a pyramid equals *one half* of the perimeters of the two bases multiplied by the slant height?

9. Regarding a frustum of a cone as a frustum of a pyramid whose lateral surface is composed of a very large

number of small trapezoids, will the rule for finding the lateral surface of a frustum of a pyramid apply in finding the lateral surface of the frustum of a cone?

10. Find the convex surface of a frustum of a square pyramid, each side of the lower base being 12 ft. and upper base 6 ft., and the slant height 75 ft.

11. Find the convex surface of a frustum of a cone whose slant height is 18 ft., the circumference of the lower base being 15 ft., and of the upper base 8 ft.

12. Find the entire surface of the frustum described in Ex. 10. Of the frustum described in Ex. 11.

LESSON 60

A **Sphere** is a solid bounded by a uniformly curved surface, every part of which is equally distant from a point within, called the center.

The **Diameter** of a sphere is a straight line passing from any point in the circumference through the center and terminating in the surface directly opposite.

The **Radius** is a straight line drawn from any point in the circumference to the center of the sphere.

SPHERE

The surface of a sphere equals that of a rectangle whose length is equal to the circumference of the sphere, and whose width is equal to the diameter of the sphere. This may be clearly illustrated by placing a sphere on the table between the palms of the hands and rolling it over once. The space passed over will be as long as the circumference

of the sphere, and the distance between the palms of the hands will be as wide as the diameter of the sphere.

1. The edge AD represents which dimension of the sphere? The edge CD represents which dimension?

2. If AB is 8 in. what is the length of BC?

3. What is the surface of the sphere?

4. Can you formulate a rule for finding the surface of a sphere?

5. The diameter of a sphere is 10 in. What is the surface?

6. What is the surface of a sphere whose diameter is 9 in.?

7. What is the surface of a sphere whose radius is $3\frac{1}{2}$ in.?

8. What is the surface of a sphere whose circumference is 17.2788 in.?

9. What is the surface of a sphere whose circumference is 31.416 in.?

10. (*a*) Circumference × diameter = what?
(*b*) Surface ÷ circumference = what?
(*c*) Surface ÷ diameter = what?

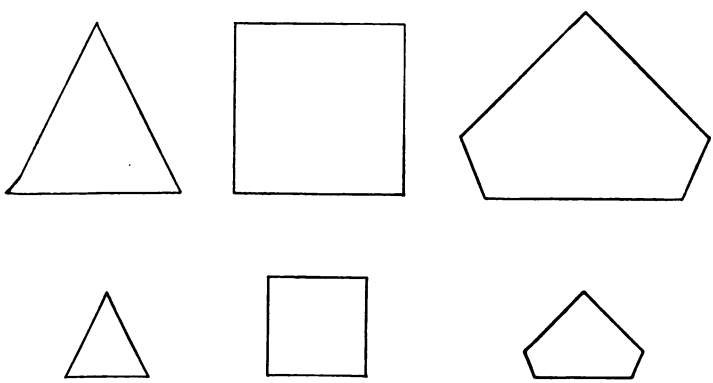
11. The surface of a sphere is 50.2656 sq. in.; the diameter is 4 in. What is the circumference?

12. The surface of a sphere is 78.54 sq. in.; the circumference 15.708 in. What is the diameter?

LESSON 61

Similar Figures are those that have the same shape, but differ in size.

To have the same shape, two figures must have corresponding equal angles, the same number of sides, and the sides containing the angles must be proportional.

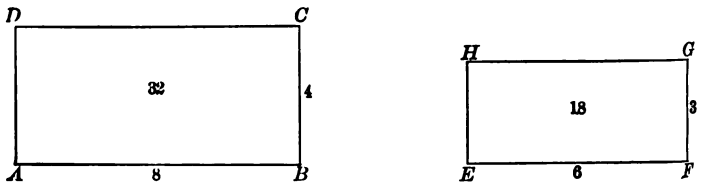


The following principles of similar figures are derived from geometry:

a. *The areas of similar figures are to each other as the squares of their like dimensions.*

b. *The like dimensions of similar figures are to each other as the square roots of their areas.*

1. The area of a rectangle whose longer side is 8 ft. is 32 sq. ft. What is the area of a similar rectangle whose longer side is 6 ft.?



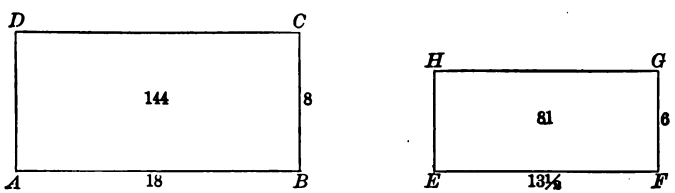
Let *ABCD* represent the larger rectangle, and *EFGH* the smaller.

From the conditions of the problem it is evident that the area *EFGH* is less than the area *ABCD*. According to Prin. *a* the area *ABCD* must sustain the same relation to the area *EFGH* that the square of the length *AB* does to the square of the length *EF*.

Hence $$8^2 : 6^2 :: 32 : x.$$

$$x = \frac{6^2 \times 32}{8^2} = 18. \quad \textit{Ans.}$$

2. The area of a rectangle whose longer side is 18 ft. is 144 sq. ft. Find the longer side of a similar rectangle whose area is 81 sq. ft.



Let *ABCD* represent the larger rectangle, and *EFGH* the smaller.

From the conditions of the problem it is evident that the length EF is not so great as the length AB.

According to Prin. *b* the length AB sustains the same relation to the length EF that $\sqrt{\text{area } ABCD}$ does to $\sqrt{\text{area } EFGH}$.

Hence $\sqrt{144} : \sqrt{81} :: 18 : x.$

Or $12 : 9 :: 18 : x.$

$$x = \frac{9 \times 18}{12} = 13\tfrac{1}{2}. \quad \textit{Ans.}$$

3. A triangle whose altitude is 8 ft. has an area of 36 sq. ft. What is the area of a similar triangle whose altitude is 12 ft.? (Prin. *a*.)

STATEMENT

$8^2 : 12^2 :: 36 : x.$

4. The diameter of a circle whose area is 78.54 sq. rd. is 10 rd. What is the diameter of a circle whose area is 490.875 sq. rd.? (Prin. *b*.)

STATEMENT

$\sqrt{78.54} : \sqrt{490.875} :: 10 : x.$

5. There are two circles, one 6 ft. in diameter, and the other 30 ft. The second is how many times as large as the first?

STATEMENT

$6^2 : 30^2 :: 1 : x.$

6. The side of a square field that contains 10 A. is 40 rd. Find the side of a similar field that will contain 30 A.

7. If a pipe 2 in. in diameter fills a cistern in 45 min., in what time will a pipe $1\frac{1}{2}$ in. in diameter fill it?

8. The area of an equilateral triangle is 20.5 sq. ft. What is the area of a similar triangle whose sides are one half as long?

9. The area of a circle whose diameter is 12 ft. is 113.0976 sq. ft. What is the diameter of a circle whose area is 12.5664 sq. ft.?

10. The area of a right-angled triangle whose perpendicular is 18 ft. is 216 sq. ft. Find the perpendicular of a similar triangle whose area is 384 sq. ft.

11. Mr. Singly has a rectangular piece of land 120 rd. long and 80 rd. wide. Find the dimensions of a similar field that will contain $93\frac{3}{4}$ A.

12. The length of a rectangular piece of land containing 1920 sq. rd. is 48 rd. Find the length of a similar piece of land containing 7680 sq. rd.

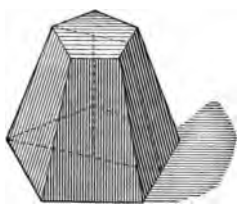
13. A man has a circular garden 11 rd. in diameter. Find the diameter of a similar piece of land 16 times as large.

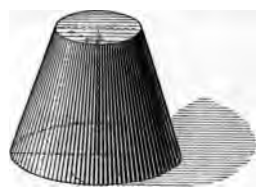
SOLIDS

LESSON 62

(See Lesson 59)

To find the volume of a frustum, take the sum of the areas of the two bases; to this add the square root of their product, and multiply this sum by one third of the altitude.





1. Find the solid contents of a frustum of a square pyramid whose altitude is 21 ft., each side of the lower base being 4 ft. and of the upper base 3 ft.

OPERATION

$4 \times 4 = 16$ = area of lower base.
$3 \times 3 = 9$ = area of upper base.
$\sqrt{16 \times 9} = 12$ = square root of their product.
$16 + 9 + 12 = 37$ = sum of the bases and square root of their product.
$37 \times (\frac{1}{3} \times 21) = 259$ cu. ft. *Ans.*

STATEMENT

$(4^2 + 3^2 + \sqrt{4^2 \times 3^2}) \times (\frac{1}{3} \times 21) =$ answer.

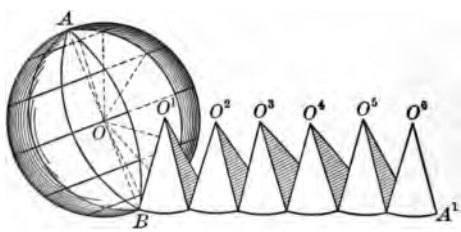
2. Find the solid contents of a frustum of a cone whose altitude is 18 ft., diameters of bases being 8 ft. and 6 ft.

3. How many cubic feet are there in 50 fence posts 8 ft. long, the larger end being 8 in. square and the smaller end 6 in. square?

4. How many cubic feet are there in a piece of timber 24 ft. long, the greater end being 12 in. square, and the less end 9 in. square?

5. Find the convex surface of a frustum of a square pyramid whose slant height is 8 ft., each side of the greater base 5 ft. and of the less base 2 ft.

LESSON 63



If we regard a sphere as composed of a very large number of pyramids, the sum of the areas of the bases of the pyramids will form the surface of the sphere, the altitude of each pyramid the radius of the sphere, and the sum of the volumes of the pyramids the volume of the sphere.

Therefore, the rule for finding the volume of a pyramid may be applied in finding the volume of a sphere.

To find the volume of a sphere multiply the surface by one third of the radius.

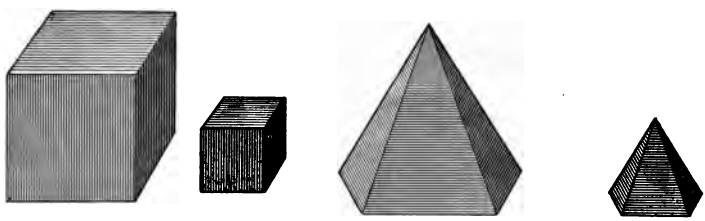
1. Find the circumference of a sphere whose diameter is 6 ft. Find the surface. Find the volume.

2. Find the volume of a sphere whose circumference is 25.1328 ft.

3. Find the difference in volume between a 4-inch cube, and a sphere having a diameter of 4 in.

LESSON 64

(See Lesson 61)



1. Compare the volume of a 2-inch cube with that of a 4-inch cube.

2. Compare the volume of a 3-inch cube with that of a 6-inch cube.

3. Multiplying the edge of a cube by 2 makes the volume how many times as great?

4. Multiplying the edge of a cube by 3 makes the volume how many times as great?

Whatever is true of the cube is true of any similar solids.

The following principles of similar solids are derived from geometry:

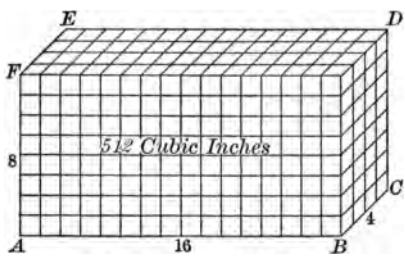
(*a*) *Similar solids are to each other as the cubes of their like dimensions.*

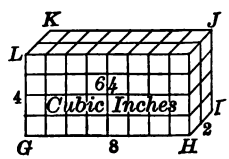
(*b*) *Like dimensions of similar solids are to each other as the cube roots of their contents.*

5. A rectangular solid 16 in. long contains 512 cu. in. How many cubic inches are contained in a similar solid 8 in. long?

Let *ABCDEF* represent the first rectangular solid, and *GHIJKL* the second.

From the conditions of the problem it is evident that the volume *GHIJKL* is less than *ABCDEF*. According to Prin. *a* the volume *ABCDEF* must sustain the same relation to volume *GHIJKL* that the cube of the length *AB* does to the cube of the length *GH*.





Hence $16^3 : 8^3 :: 512 : x.$

$$x = \frac{8^3 \times 512}{16^3} = 64 \text{ } Ans.$$

$$2^3$$

6. A rectangular solid 16 in. long contains 512 cu. in. Find the length of a similar solid that contains 64 cu. in.

Using the same figures as in the first illustrative example, it is plain that the length *GH* is not so great as the length *AB*.

According to Prin. *b* the length *AB* sustains the same relation to *GH* that the cube root of the volume *ABCDEF* does to the cube root of volume *GHIJKL*.

Hence $\sqrt[3]{512} : \sqrt[3]{64} :: 16 : x.$

$$x = \frac{\sqrt[3]{64} \times 16}{\sqrt[3]{512}} = 8 \text{ } Ans.$$

$$\sqrt[3]{8}$$

7. A ball 6 inches in diameter is how many times as large as one 3 inches in diameter?

STATEMENT

$3^3 : 6^3 :: 1 : x.$

8. If a 3-inch cube weighs 27 lb., how much will a 7-inch cube of the same material weigh?

STATEMENT

$3^3 : 7^3 :: 27 : x.$

9. If a 4-inch cube weighs 64 lb., what will be the length of the edge of a cube that weighs 216 lb.?

STATEMENT

$\sqrt[3]{64} : \sqrt[3]{216} :: 4 : x.$

10. If a ball of metal 6 in. in diameter weighs 135 lb., what is the diameter of a ball of the same metal whose weight is 5 lb.?

11. A cylinder whose diameter is 4 ft. contains 125 cu. ft. Find the diameter of a cylinder that contains 512 cu. ft.

12. A cone 8 ft. high contains 100 cu. ft. What is the altitude of a cone that contains 50 cu. ft.?

13. A pyramid 6 in. high weighs 32 lb. What is the altitude of a pyramid that weighs 256 lb.?

14. A stack of hay 20 ft. in diameter contains 5 tons. How many tons are there in a stack 12 ft. in diameter?

REVIEW WORK

LESSON 65

1. Find the surface of a 6-inch cube.

2. Find the surface of a rectangular solid 8 ft. long, 4 ft. wide, and 3 ft. thick.

3. How many cubic feet of earth must be removed in digging a well 6 ft. in diameter and 30 ft. deep?

4. The base of a right-angled triangle is 60 ft., and the perpendicular 80 ft. Find the hypotenuse.

5. Find the length of the longest stick that can be placed within the well described in Ex. 3.

6. Find the side of a square the area of which is equal to the area of a rectangle 280 ft. long and 70 ft. wide.

7. The floor of a room 24 ft. long contains $42\frac{2}{3}$ sq. yd. Find the width.

8. A farmer took 2200 steps of 3 ft. each to walk around a rectangular field. How many acres are there in the field, if its length is to its width as 2 to 1?

9. What is the height of a room 13 ft. long and 12 ft. wide, if it contains 2,695,680 cu in.?

10. Find the side of the largest square that can be drawn within a circle 20 in. in diameter.

11. How many gallons of water will a cylindrical cistern hold, if it is 6 ft. 4 in. deep and 4 ft. 3 in. in diameter?

12. Find the area of a square that can be drawn within a circle whose radius is 8 ft.

STATEMENT

$(8 \times 2)^2 \div 2 =$ answer.

NOTE. — The diameter of a circle equals the diagonal of the inscribed square. To find the area of a square when the diagonal is given, divide the *square* of the diagonal by 2.

13. Find the largest square stick of timber that can be cut from a log 16 in. in diameter.

14. Find the cost of fencing a circular driving park 40 rd. in diameter, at $1.25 a rod.

15. The volume of a cone is 384 cu. in. The area of the base is 64 sq. in. Find the altitude.

16. How many gallons of water will a tub hold 18 in. deep, 26 in. wide at the top, and 20 in. at the bottom?

17. A pipe $1\frac{1}{2}$ in. in diameter will fill a cistern with water in $1\frac{1}{2}$ hr. In what time will a pipe 2 in. in diameter fill it?

LESSON 66

1. The length of a rectangular field containing 30 acres is to its breadth as 8 to 6. Find its dimensions.

2. A ball of brass 4 in. in diameter weighs 9 lb. What will be the diameter of a ball of the same material which weighs 576 lb.?

3. Find the volume of a pyramid whose base is a rectangle 6 ft. by 8 ft., and whose edges which meet at the apex are 14 ft.

4. Find the entire surface of the figure described in Ex. 3.

5. Find the cost of digging a ditch, at 20¢ a cubic yard, 300 rd. long, 20 in. deep, 18 in. wide at the bottom, and 2 ft. wide at the top.

6. Find the value of a piece of timber 20 ft. long, 12 in. square at one end, and 8 in. square at the other, at 22¢ a cubic foot.

7. Find the distance from one of the lower corners of a room to the opposite upper corner, if the room is 30 ft. long, 20 ft. wide, and 12 ft. high.

8. How many spheres 3 in. in diameter can be made from 2 spheres 6 in. in diameter?

9. A building 80 ft. long and 60 ft. wide has a pyramidal roof 15 ft. high. Find the length of the rafters reaching from the corners to the highest point of the roof.

10. How much, at 4¢ a square foot, will it cost to cover with tin the roof described in Ex. 9?

11. How many cones 3 ft. in diameter and 8 ft. high can be made from a cylinder of the same diameter and height?

12. Find the slant height of a pyramid whose base is 4 ft. square and altitude 10 ft.

13. Find the entire surface of a cube containing 5832 cu. ft.

LESSON 67

1. The premium for insuring my house at $2\frac{1}{2}\%$ was \$90. For what sum was it insured?

2. I bought 50 shares of stock at 105, and sold them at $1\frac{1}{2}\%$ discount. Find the loss.

3. The amount of \$1600 at a certain rate for 2 yr. 6 mo. is \$1880. Find the rate.

4. What part of a mile is 120 rd. 3 yd. 2 ft.?

5. If I buy 300 pears at the rate of 2 for 1¢, and 270 at the rate of 3 for 1¢, and sell them all at the rate of 5 for 2¢, shall I gain or lose, and how much?

6. Draw on paper, on a scale of $\frac{1}{8}$ in. to the foot, a diagram of a rectangular flower bed 56 ft. by 48 ft., with a walk 3 ft. wide around the outside of it. Find the length of the entire figure. The width. The perimeter.

7. How many square yards are there in the flower bed?

8. Find the perimeter of the outside of the walk in feet. In yards.

9. The inside perimeter of the walk is how much less than the outside perimeter?

10. How many square yards are there in the walk?

11. My agent charged me $\frac{1}{2}\%$ commission for selling flour. His commission was $216. How many dollars worth of flour did he sell? If he sold it at $5.40 a barrel, how many barrels did he sell? How many dollars did I receive for my flour?

LESSON 68

1. Moving the decimal point two places to the right has what effect on the value of the decimal?

2. What part of $\frac{2}{3}$ is $\frac{3}{4}$? What is the ratio of $\frac{3}{4}$ to $\frac{2}{3}$?

3. 20 square rods equals what decimal of an acre?

4. Dividing the denominator of a fraction has what effect on the value of the fraction?

5. What is ratio? Greatest common divisor of several numbers?

6. Find the amount of $700 from Nov. 13, 1901, to June 1, 1902, at 5%.

7. A cistern 8 ft. × 4 ft. × 3 ft. holds how many barrels of water?

8. Find the diagonal of a square piece of land containing 40 acres.

9. Multiply $\frac{3}{5}$ by $\frac{5}{7}$ and explain why canceling the 5 in the numerator and denominator will give the correct result.

10. Find the solid contents and surface of a sphere 10 in. in diameter.

11. Find the proceeds of the following note:

750\frac{00}{100}$. BUFFALO, Feb. 13, 1902.

Six months after date, for value received, I promise to pay John Sheldon, or order, Seven Hundred Fifty Dollars, with interest at 6%. WILLIAM KINEY.

Discounted at 6% May 13.

12. Who is the maker of the above note? The payee?

LESSON 69

1. Find the entire surface of a cone whose altitude is 8 ft. and diameter of base 12 ft.

2. The principal is $480; interest, $64.08; time, 2 yr. 11 mo. 18 da. Find the rate.

3. What is the ratio of 3 rd. 4 yd. to 5 rd. 1 ft.?

4. Name two numbers whose ratio equals the ratio of $\frac{2}{3}$ to $\frac{3}{5}$.

5. Name two fractions whose ratio equals the ratio of 8 to 9.

6. A man sold two cows for $120; on one the gain was 20%, and on the other the loss was 20%. Find the net gain or loss.

7. What is the ratio of the volume of a sphere 6 in. in diameter to that of a cone whose altitude is 6 in., and diameter at the base 6 in.?

8. Reduce $2800 to English money.

9. James and Henry have $330. How much money has each if $\frac{2}{3}$ of James's equals $\frac{4}{5}$ of Henry's?

10. If 17 bu. 37 lb. of corn cost $8.75, how much will 26 bu. cost?

11. How long is the day when the sun rises 21 min. before 5, and sets 20 min. after 7?

12. Express as per cents $\frac{5}{8}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{9}$.

13. Compare the area of a circle whose circumference is 25.1328 ft., with the surface of a sphere whose circumference is 25.1328 ft.

14. Is it true that the surface of a sphere is exactly equal to the areas of four circles, each having the same circumference as the sphere?

THE METRIC SYSTEM

LESSON 70

The **Metric System** is a system of weights and measures based upon the decimal plan of notation. It originated in France about 1795. It has been adopted, or its use is recommended, wholly or in part, by nearly all civilized countries. It is called the *metric system* because the *meter* is taken as the fundamental unit. The length of the meter is about the one ten-millionth part of the distance on the earth from the equator to the north pole, or 39.37 inches.

One of the commendable features of the *metric system* is its simplicity. When the unit of measure has been decided upon, all denominations ascend and descend regularly in a tenfold ratio; that is, they are formed by multiplying or by dividing by 10. For example, if we take the meter as the unit of measure, the denominations above the meter are formed by prefixing the words *deka*, *hekto*, *kilo*, and *myria* to the primary unit — dekameter, meaning 10 meters; hektometer, meaning 100 meters; kilometer, meaning 1000 meters; and myriameter, meaning 10,000 meters.

The denominations below the meter are formed by prefixing the abbreviations deci, centi, and milli to the unit — decimeter, meaning $\frac{1}{10}$ of a meter; centimeter, $\frac{1}{100}$ of a meter; and millimeter, $\frac{1}{1000}$ of a meter.

NOTE. — Deka, hekto, kilo, and myria are Greek numerals, meaning respectively 10, 100, 1000, and 10,000. Deci, centi, and milli are Latin abbreviations, meaning respectively $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1000}$.

MEASURES OF LENGTH

The unit of **Length** is the *meter*. It is about 39.37 in. or 3.28 ft. in length.

TABLE

10 millimeters (mm)	= 1 centimeter (cm).
10 centimeters	= 1 decimeter (dm).
10 decimeters	= 1 *meter* (m).
10 meters	= 1 dekameter (Dm).
10 dekameters	= 1 hektometer (Hm).
10 hektometers	= 1 kilometer (Km).
10 kilometers	= 1 myriameter (Mm).

EQUIVALENTS

1 centimeter = .3937079 in.	1 dekameter = 32.80899 ft.
1 decimeter = 3.937079 in.	1 hektometer = 19.92781 rd.
1 meter = 39.37079 in.	1 kilometer = .621382 mi.

1 myriameter = 6.21382 mi.

Very long distances are measured by the kilometer; very short distances by the millimeter, and intermediate distances by the meter.

The change from one denomination to another of equal value is effected by changing the position of the decimal point. To change from a lower to a higher denomination, we move the decimal point to the left. For example, 1456.7 meters = 145.67 dekameters = 14.567 hektometers = 1.4567 kilometers.

To change from a higher to a lower denomination, we move the decimal point to the right. For example, 86.456 meters = 864.56 decimeters = 8645.6 centimeters = 86456 millimeters.

Metric numbers involving integers and decimals are read just as integral and decimal denominate numbers are read. Thus, 12.35^{m} is read twelve and thirty-five hundredths meters.

Figures used to represent decimal measures may be written in terms of any denomination. Thus, $7^{m}\,4^{dm}\,8^{cm}\,5^{mm}$ may be written as 7.485^{m}, or 74.85^{dm}, or 748.5^{cm}, or 7485^{mm}.

1. How many decimeters in 4 meters? In $5\frac{1}{2}$ meters?
2. How many centimeters in 3 meters and 5 decimeters?
3. How many dekameters in 456 meters?
4. How many decimeters in 3869 millimeters?
5. How many dekameters in $3\frac{1}{2}$ kilometers?
6. How many meters in $5^{Dm}\,2^{m}\,4^{dm}\,3^{cm}$?
7. How much will $5^{m}\,8^{dm}$ of ribbon cost at 20¢ a meter?
8. How many centimeters in $3^{m}\,4^{dm}$?
9. If $4^{Dm}\,3^{m}$ of silk cost $86, find the cost of 1 meter.
10. Find the cost of $2^{Dm}\,3^{m}\,5^{dm}$ of cassimere at 50¢ a meter.
11. Reduce $9^{m}\,8^{cm}\,7^{mm}$ to millimeters.
12. Reduce $8^{Dm}\,7^{m}$ to centimeters.
13. Express 1463.24^{Dm} as kilometers; as meters.
14. Write 3684.9^{dm} as dekameters; as centimeters.
15. Change $8^{Km}\,4^{Hm}\,8^{m}\,7^{dm}$ to meters.
16. Add 7.5^{Dm} 5^{dm} 4054^{mm} and 12^{m}. Answer in meters.
17. Find the difference between 28.46^{m} and 216^{dm}. Answer in meters.
18. Multiply 92.5^{cm} by 24. Answer in meters.
19. A merchant bought 4 pieces of dress goods, each piece containing $5^{Dm}\,4^{m}\,5^{dm}$, at 80¢ a meter. Find the cost.
20. A man paid $179.20 for $8^{Dm}\,9^{m}\,6^{dm}$ of cloth. How much was that per meter?

LESSON 71

Measures of Surface

The unit of **Surface Measure** is the *square meter*. It is a square each side of which is a meter in length.

Let us suppose that Fig. 1 represents a square meter, and Fig. 2 a square decimeter.

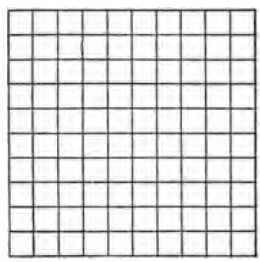


Fig. 1

Fig. 2

Since the meter equals 10 decimeters, it is evident that each side of Fig. 1 is 10 decimeters in length, and it follows that the square meter contains 10×10 or 100 square decimeters. Again, since each decimeter equals 10 centimeters, a square decimeter equals 10×10, or 100 square centimeters. Also, as the centimeter equals 10 millimeters, the square centimeter equals 10×10, or 100 square millimeters.

Hence the following table :

Table

100 square millimeters (sq mm)	= 1 square centimeter (sq cm)
100 square centimeters	= 1 square decimeter (sq dm)
100 square decimeters	= 1 *square meter* (sq m)
100 square meters	= 1 square dekameter (sq Dm)
100 square dekameters	= 1 square hektometer (sq Hm)
100 square hektometers	= 1 square kilometer (sq Km)

Equivalents

1 sq. centimeter	= .155+ sq. in.	1 sq. dekameter	= 119.6034 sq. yd.
1 sq. decimeter	= 15.5+ sq. in.	1 sq. hektometer	= 2.47114 A.
1 sq. meter	= 1.196+ sq. yd.	1 sq. kilometer	= 247.114 A.
			= .3861 sq. mi.

In measuring land, only three of the preceding denominations are used. These are the square meter, called a *centare* (ca), the square dekameter, called an *are* (a), and the square hektometer, called a *hektare* (Ha).

NOTE. — Since it takes 100 of one denomination to make one of the next higher in measures of surface, the decimal point must be moved two places for every interval, in changing from one denomination to another.

1. How many hektares are there in 38,649 ares?

2. How many square centimeters are there in 89,645 square decimeters?

3. Express $22.14^{\text{sq Dm}}$ as square meters; as square centimeters.

4. Express $89{,}634^{\text{sq m}}$ as square millimeters; as square dekameters.

5. Find the cost of $14^{\text{Ha}}\ 25^{\text{a}}$ of land, at $56 per hektare.

6. How many hektares are there in $53{,}684^{\text{a}}$? How many ares in 38.645^{ca}?

7. How many square dekameters are there in $89{,}645^{\text{sq m}}$?

8. How many centares are there in a floor 22.3^{m} long and 18.8^{m} wide?

9. A man bought 6800 ares of woodland at $5 an are, and sold it at $550 a hektare. Find the gain.

10. A rectangular piece of land containing 1028.31^{ca} is 45.3^{m} long. What is its width?

11. How many square yards are there in 427.5 ares?

12. How many ares are there in 1080 square yards?

13. How many acres are there in 1516.8 ares?

14. How many ares are there in 90 acres?

15. I have a room 10.6 meters long and 8.4 meters wide. How much carpet a meter wide will be required to cover the floor?

LESSON 72

Measures of Volume

In **Solid** or **Cubic Measure** the *cubic meter* is regarded as the unit.

Let us suppose that Fig. 1 represents a cubic meter, and Fig. 2 a cubic decimeter.

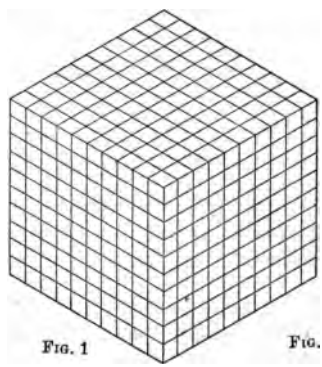


Fig. 1

Fig. 2

Since the meter equals 10 decimeters, it is evident that each edge of Fig. 1 is 10 decimeters in length, and it follows that the cubic meter contains $10 \times 10 \times 10$, or 1000 cubic decimeters. Since each decimeter equals 10 centimeters, the cubic decimeter equals $10 \times 10 \times 10$, or 1000 cubic centimeters. Also as the centimeter equals 10 millimeters, the cubic centimeter equals $10 \times 10 \times 10$, or 1000 cubic millimeters.

Hence the following table:

Table

1000 cubic millimeters (cu mm)	= 1 cubic centimeter (cu cm)
1000 cubic centimeters	= 1 cubic decimeter (cu dm)
1000 cubic decimeters	= 1 *cubic meter* (cu m)

Equivalents

1 cu. centimeter	= .061+ cu. in.
1 cu. decimeter	= 61.026+ cu. in.
1 cu. meter	= 35.316+ cu. ft., or 1.308 cu. yd.

The denominations above the cubic meter are not generally used. In expressing the contents of all ordinary solids the cubic meter is employed. In measuring wood, and lumber in piles, the cubic decimeter, cubic meter, and cubic dekameter are employed.

For the sake of brevity the cubic meter is generally called a *stere* (st); the cubic decimeter a *decistere* (dst); and the cubic dekameter a *dekastere* (Dst).

TABLE OF WOOD MEASURE

10 decisteres (dst) = 1 *stere* (st)
10 steres = 1 dekastere (Dst)

EQUIVALENTS

1 decistere = 3.531+ cu. ft.
1 stere = 35.316+ cu. ft.
1 dekastere = 13.079+ cu. yd.

NOTE. — Since it takes 1000 of one denomination to make one of the next higher in cubic measure, the decimal point must be moved *three* places for every interval in changing from one denomination to another.

1. How many cubic millimeters are there in $8\frac{1}{2}^{\text{cu cm}}$? In $4\frac{1}{2}^{\text{cu dm}}$?

2. How many cubic meters are there in $95{,}600^{\text{cu dm}}$?

3. Write $421.372^{\text{cu m}}$ as cubic decimeters; $75{,}006^{\text{cu dm}}$ as steres.

4. Write $136.84^{\text{cu m}}$ as cubic dekameters; as cubic decimeters.

5. Reduce $7^{\text{st}}\ 850^{\text{dst}}$ to cubic meters.

6. Find the cost of 14 steres 6 decisteres of wood at $\$1.87$ a stere.

7. If $7^{\text{Dst}}\ 5^{\text{st}}$ of oak plank cost $\$110.625$, how much will $3^{\text{Dst}}\ 7^{\text{st}}\ 5^{\text{dst}}$ cost?

8. If it costs \$45.12 to remove $112^{st}\ 8^{dst}$ of earth, how much will it cost to remove $21\frac{3}{5}^{st}$?

LESSON 73

Measures of Capacity

The unit of **Capacity** is the *liter*. It is used for both liquid and dry measure. It is equal to a cubical vessel whose edge is a decimeter, or 3.937079 in.

Table

10 milliliters (ml)	= 1 centiliter (cl)
10 centiliters	= 1 deciliter (dl)
10 deciliters	= 1 *liter* (l)
10 liters	= 1 dekaliter (Dl)
10 dekaliters	= 1 hektoliter (Hl)
10 hektoliters	= 1 kiloliter (Kl)

Equivalents

Denominations	Dry Measure	Liquid Measure
1 centiliter	= .6102 cu. in.	= .338 fluid oz.
1 deciliter	= 6.1022 cu. in.	= .845 gill
1 *liter*	= .908 qt.	= 1.0567 qt.
1 dekaliter	= 9.08 qt.	= 2.6417 gal.
1 hektoliter	= 2.8372+ bu.	= 26.417 gal.
1 kiloliter	= 28.372+ bu.	= 264.17 gal.

The liter is used chiefly in measuring liquids, and the hektoliter in measuring dry substances.

1. Change 7.36^{l} to deciliters; to milliliters.

2. Express $9^{Dl}\ 7^{l}\ 2^{cl}$ as liters; as centiliters.

3. Write 5386^{cl} as liters; as hektoliters.

4. Change 72.3467^{Kl} to liters; to deciliters; to milliliters.

5. Add 7.3^{l}, 5^{dl}, 29^{cl}, and 274^{ml}.

6. Multiply 6.48^{cl} by 36. Divide 6.48^{cl} by 8.

7. Find the cost of 7 dekaliters 4 liters of currants, at 12¢ a liter.

8. Find the sum of 3 hektoliters 4 dekaliters 7 liters, 5 hektoliters 4 liters, 9 dekaliters 4 liters.

9. Find the cost of 5 hektoliters 3 dekaliters of corn, at $1.80 a hektoliter.

10. At 62¢ a liter, find the cost of 9 liters 8 deciliters of molasses.

LESSON 74

Measures of Weight

The unit of **Weight** is the *gram*. It equals 15.432 Troy grains.

Table

10 milligrams (mg)	= 1 centigram (cg)
10 centigrams	= 1 decigram (dg)
10 decigrams	= 1 *gram* (g)
10 grams	= 1 dekagram (Dg)
10 dekagrams	= 1 hektogram (Hg)
10 hektograms	= 1 kilogram (Kg)
10 kilograms	= 1 myriagram (Mg)
10 myriagrams	= 1 quintal (Q)
10 quintals	= 1 tonneau, or ton (T)

Equivalents

1 centigram = .1543+ Troy gr.	1 hektogram = 3.5274 oz. Av.
1 decigram = 1.5432+ Troy gr.	1 kilogram or kilo = 2.6792+ lb. Troy, 2.2046+ lb. Av.
1 gram = 15.432+ Troy gr.	1 myriagram = 22.046+ lb. Av.
1 dekagram = .3527+ oz. Av.	1 quintal = 220.46+ lb. Av.

1 tonneau, or ton = 2204.62+ lb. Av., or 1.1023+ tons.

The gram and its subdivisions are used for weighing very small articles, letters, medicines, gold, silver, and for philosophical experiments.

For ordinary business purposes the kilogram, or kilo, is probably more frequently employed than any other denomination. It is about $2\frac{1}{5}$ lb. Av.

The quintal and the tonneau are principally used for weighing heavy articles, such as iron, hay, etc.

The quintal = 100 kilos, or about 220.46 lb. Av. The tonneau = 1000 kilos, or about 2204.6 lb.

The hektogram is very frequently written simply *hekto.*

1. Express $296{,}431^{cg}$ as grams; as hektograms; as kilograms.

2. Express 76.34^{Kg} as tons; as hektograms; as grams.

3. Write $7^{Kg}\ 5^{Dg}\ 7^{g}\ 5^{cg}$ and 4^{mg} as grams.

4. How many grams in 560 grains?

5. How many grams in 5 lb. Av.?

6. How many pounds Av. in 536 kilos?

7. Find the sum of $7^{Dg}\ 6^{g}\ 8^{dg}$, $3^{Dg}\ 5^{dg}$, $8^{g}\ 6^{cg}\ 4^{dg}$. Answer in grams.

8. Multiply 36.7 grams by 65. Answer in hektograms.

9. At 8¢ a gram, what would be the cost of 25 grams 8 decigrams of quinine?

10. How many letters each weighing 3.5^{g} will be required to weigh 1.75^{Kg}?

11. If a car weighs 28,000 pounds, what is its weight in metric tons?

12. A barrel of flour weighs 196 lb. Express its weight in metric numbers.

Metric numbers involving integers and decimals are read just as integral and decimal denominate numbers are read. Thus, 12.35^{m} is read twelve and thirty-five hundredths meters.

Figures used to represent decimal measures may be written in terms of any denomination. Thus, $7^{m}\,4^{dm}\,8^{cm}\,5^{mm}$ may be written as 7.485^{m}, or 74.85^{dm}, or 748.5^{cm}, or 7485^{mm}.

1. How many decimeters in 4 meters? In $5\frac{1}{2}$ meters?
2. How many centimeters in 3 meters and 5 decimeters?
3. How many dekameters in 456 meters?
4. How many decimeters in 3869 millimeters?
5. How many dekameters in $3\frac{1}{2}$ kilometers?
6. How many meters in $5^{Dm}\,2^{m}\,4^{dm}\,3^{cm}$?
7. How much will $5^{m}\,8^{dm}$ of ribbon cost at 20¢ a meter?
8. How many centimeters in $3^{m}\,4^{dm}$?
9. If $4^{Dm}\,3^{m}$ of silk cost \$86, find the cost of 1 meter.
10. Find the cost of $2^{Dm}\,3^{m}\,5^{dm}$ of cassimere at 50¢ a meter.
11. Reduce $9^{m}\,8^{cm}\,7^{mm}$ to millimeters.
12. Reduce $8^{Dm}\,7^{m}$ to centimeters.
13. Express 1463.24^{Dm} as kilometers; as meters.
14. Write 3684.9^{dm} as dekameters; as centimeters.
15. Change $8^{Km}\,4^{Hm}\,8^{m}\,7^{dm}$ to meters.
16. Add $7.5^{Dm}\,5^{dm}\,4054^{mm}$ and 12^{m}. Answer in meters.
17. Find the difference between 28.46^{m} and 216^{dm}. Answer in meters.
18. Multiply 92.5^{cm} by 24. Answer in meters.
19. A merchant bought 4 pieces of dress goods, each piece containing $5^{Dm}\,4^{m}\,5^{dm}$, at 80¢ a meter. Find the cost.
20. A man paid \$179.20 for $8^{Dm}\,9^{m}\,6^{dm}$ of cloth. How much was that per meter?

LESSON 71

Measures of Surface

The unit of **Surface Measure** is the *square meter*. It is a square each side of which is a meter in length.

Let us suppose that Fig. 1 represents a square meter, and Fig. 2 a square decimeter.

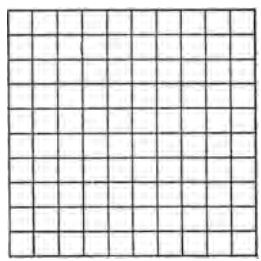


Fig. 1

Fig. 2

Since the meter equals 10 decimeters, it is evident that each side of Fig. 1 is 10 decimeters in length, and it follows that the square meter contains 10×10 or 100 square decimeters. Again, since each decimeter equals 10 centimeters, a square decimeter equals 10×10, or 100 square centimeters. Also, as the centimeter equals 10 millimeters, the square centimeter equals 10×10, or 100 square millimeters.

Hence the following table:

Table

100 square millimeters (sq mm)	= 1 square centimeter (sq cm)
100 square centimeters	= 1 square decimeter (sq dm)
100 square decimeters	= 1 *square meter* (sq m)
100 square meters	= 1 square dekameter (sq Dm)
100 square dekameters	= 1 square hektometer (sq Hm)
100 square hektometers	= 1 square kilometer (sq Km)

Equivalents

1 sq. centimeter	= .155+ sq. in.	1 sq. dekameter	= 119.6034 sq. yd.
1 sq. decimeter	= 15.5+ sq. in.	1 sq. hektometer	= 2.47114 A.
1 sq. meter	= 1.196+ sq. yd.	1 sq. kilometer	= 247.114 A.
			= .3861 sq. mi.

Metric numbers involving integers and decimals are read just as integral and decimal denominate numbers are read. Thus, 12.35^{m} is read twelve and thirty-five hundredths meters.

Figures used to represent decimal measures may be written in terms of any denomination. Thus, $7^{m}\,4^{dm}\,8^{cm}\,5^{mm}$ may be written as 7.485^{m}, or 74.85^{dm}, or 748.5^{cm}, or 7485^{mm}.

1. How many decimeters in 4 meters? In $5\frac{1}{2}$ meters?
2. How many centimeters in 3 meters and 5 decimeters?
3. How many dekameters in 456 meters?
4. How many decimeters in 3869 millimeters?
5. How many dekameters in $3\frac{1}{2}$ kilometers?
6. How many meters in $5^{Dm}\,2^{m}\,4^{dm}\,3^{cm}$?
7. How much will $5^{m}\,8^{dm}$ of ribbon cost at 20¢ a meter?
8. How many centimeters in $3^{m}\,4^{dm}$?
9. If $4^{Dm}\,3^{m}$ of silk cost \$86, find the cost of 1 meter.
10. Find the cost of $2^{Dm}\,3^{m}\,5^{dm}$ of cassimere at 50¢ a meter.
11. Reduce $9^{m}\,8^{cm}\,7^{mm}$ to millimeters.
12. Reduce $8^{Dm}\,7^{m}$ to centimeters.
13. Express 1463.24^{Dm} as kilometers; as meters.
14. Write 3684.9^{dm} as dekameters; as centimeters.
15. Change $8^{Km}\,4^{Hm}\,8^{m}\,7^{dm}$ to meters.
16. Add $7.5^{Dm}\,5^{dm}\,4054^{mm}$ and 12^{m}. Answer in meters.
17. Find the difference between 28.46^{m} and 216^{dm}. Answer in meters.
18. Multiply 92.5^{cm} by 24. Answer in meters.
19. A merchant bought 4 pieces of dress goods, each piece containing $5^{Dm}\,4^{m}\,5^{dm}$, at 80¢ a meter. Find the cost.
20. A man paid \$179.20 for $8^{Dm}\,9^{m}\,6^{dm}$ of cloth. How much was that per meter?

LESSON 71

Measures of Surface

The unit of **Surface Measure** is the *square meter.* It is a square each side of which is a meter in length.

Let us suppose that Fig. 1 represents a square meter, and Fig. 2 a square decimeter.

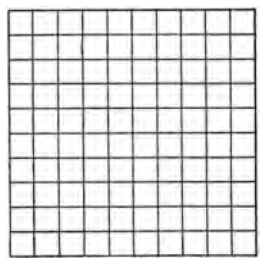


Fig. 1

Fig. 2

Since the meter equals 10 decimeters, it is evident that each side of Fig. 1 is 10 decimeters in length, and it follows that the square meter contains 10×10 or 100 square decimeters. Again, since each decimeter equals 10 centimeters, a square decimeter equals 10×10, or 100 square centimeters. Also, as the centimeter equals 10 millimeters, the square centimeter equals 10×10, or 100 square millimeters.

Hence the following table :

Table

100 square millimeters (sq mm)	= 1 square centimeter (sq cm)
100 square centimeters	= 1 square decimeter (sq dm)
100 square decimeters	= 1 *square meter* (sq m)
100 square meters	= 1 square dekameter (sq Dm)
100 square dekameters	= 1 square hektometer (sq Hm)
100 square hektometers	= 1 square kilometer (sq Km)

Equivalents

1 sq. centimeter	= .155+ sq. in.	1 sq. dekameter	= 119.6034 sq. yd.
1 sq. decimeter	= 15.5+ sq. in.	1 sq. hektometer	= 2.47114 A.
1 sq. meter	= 1.196+ sq. yd.	1 sq. kilometer	= 247.114 A.
			= .3861 sq. mi.

MISCELLANEOUS PROBLEMS

WORK PROBLEMS

LESSON 76

1. If A can mow a field in 8 da., and B can mow it in 6 da., how long will it take them both to mow it?

SOLUTION. — Since A can mow the field in 8 days, in 1 day he can mow $\frac{1}{8}$ of it; since B can mow it in 6 days, in 1 day he can mow $\frac{1}{6}$ of it; hence both can mow $\frac{1}{8} + \frac{1}{6}$, or $\frac{7}{24}$ of the field in 1 day. Since both can mow $\frac{7}{24}$ in 1 day, it will take them as many days to mow the whole field as $\frac{7}{24}$ is contained times in 1, or $3\frac{3}{7}$ days.

2. A cistern can be filled by a pipe in 3 hr. and by another in 4 hr. How long will it take both together to fill it?

3. A can do a piece of work in 6 da., B in 8 da., and C in 12 da. How long will it take them to do it together?

4. William can do a piece of work in $3\frac{1}{2}$ hr., and John can do the same work in $2\frac{1}{4}$ hr. How long will it take both together to do it?

5. B and C can build a wall in 12 da. How long will it take B alone to build the wall, if C can build it in 20 da.?

6. James can build a fence in $2\frac{1}{2}$ da. which he and Henry can build in $1\frac{1}{3}$ da. In what time can Henry build it?

7. Robert and his father can do a piece of work in 12 da. If Robert leaves at the end of 6 da. his father can finish the work in 12 da. In what time can each alone do the work?

8. Thomas can dig a ditch in 10 da., Thomas and Lewis can do the same work in 7 da., and Thomas and Irwin can do it in 6 da. In how many days can Lewis and Irwin together do it?

LESSON 77

1. If Ernest can do a piece of work in $\frac{1}{3}$ of a day, how many times the work can he do in 1 da.? If Roy can do the same work in $\frac{1}{4}$ of a day, how many times the work can he do in 1 da.? How many times the work can they both do in 1 da.? In what time can they together do the work?

2. If A can build a boat in $\frac{1}{4}$ of a month, and B can build it in $\frac{1}{6}$ of a month, in what time can both build it?

3. Mr. Johnson can make a gate in $\frac{3}{5}$ of a day. What part of the gate can he make in $\frac{1}{5}$ of a day? What part in 1 da.?

4. Mildred can make a shirt waist in $\frac{3}{5}$ of a day and Gertrude in $\frac{3}{8}$ of a day. In what time can they make it working together?

5. Morris and Willard can dig a cellar in 7 da., Morris and Walter can dig it in 6 da., and Morris can dig it in 10 da. In what time can Willard and Walter dig it together?

6. A and B can cut a field of grass in 12 hr., B and C in 9 hr., and C in 18 hr. In what time can all three do it working together, and in what time can each one do it alone?

7. A does $\frac{3}{11}$ of a piece of work in 3 da., when B joins him and they together finish the work in $2\frac{2}{3}$ da. How long would it take B alone to do the work?

TIME PROBLEMS

LESSON 78

1. What time after 1 o'clock will the hour and minute hands of a clock be exactly together?

SOLUTION. — It will be seen that the minute hand passes over 12 five-minute spaces while the hour hand passes over 1 space. The minute hand therefore gains on the hour hand 11 spaces in going 12, and to gain 1 space, the distance the hands are apart at 1 o'clock, the minute hand must go $\frac{1}{11}$ of 12 spaces, or $1\frac{1}{11}$ spaces, which equal $5\frac{5}{11}$ min. Therefore the hands will be together $5\frac{5}{11}$ min. past 1 o'clock.

2. At what time after 2 o'clock will the hour and minute hands of a clock be together? After 3 o'clock? After 4 o'clock?

3. At what time between 8 and 9 o'clock are the hands of a clock together?

4. The hour and minute hands of a clock form an angle of 60° at 2 o'clock. What time after 2 o'clock will the hands make an angle of 60°?

5. The hour and minute hands of a clock are at right angles at 3 o'clock. What time after 3 o'clock are they at right angles?

6. What time after 1 o'clock will the hour and minute hands be opposite each other?

7. The hour and minute hands make an angle of how many degrees at 4 o'clock?

8. What time after 4 o'clock will the hour and minute hands make an angle of 120°?

9. Find when first after 5 o'clock the hour and minute hands are at right angles.

TEMPERATURE

LESSON 79

A **Thermometer** is an instrument used for measuring temperatures.

Two important points are to be determined in the graduation of a thermometer — the boiling point and the freezing point of water.

In the **Fahrenheit** scale the freezing point is 32°, and the boiling point 212°. The space between these fixed points is divided into 180 equal parts, called degrees.

In the **Centigrade** scale the freezing point is marked 0°, and the boiling point 100°. The intervening space is divided into 100 equal parts, called degrees.

Since 100° C. equal 180° F., 1° C. equals 1.8° F., and since 0° C. corresponds to 32° F., we may change Centigrade readings to Fahrenheit readings by multiplying the number of Centigrade degrees by 1.8 and adding 32°.

Likewise, we may change Fahrenheit degrees to Centigrade degrees by subtracting 32° from the number of Fahrenheit degrees and dividing the remainder by 1.8.

1. Express 60° C. in the Fahrenheit scale.
2. Express 72° C. in the Fahrenheit scale.
3. Express 85° F. in the Centigrade scale.
4. Express 72° F. in the Centigrade scale.
5. If the difference between the temperatures of two bodies is 40 Fahrenheit degrees, express the difference in Centigrade degrees.
6. If the difference between the temperatures of two bodies is 36 C. degrees, express the difference in F. degrees.

SPECIFIC GRAVITY

LESSON 80

The **Specific Gravity** of a substance is its weight compared with the weight of an equal volume of some other substance taken as a standard. The standard for solids and liquids is distilled water at its maximum density (4° C. or 39.2° F.).

For example, if a piece of stone is 2.8 times as heavy as the same volume of water, its specific gravity is 2.8.

The following table gives the specific gravity of a number of substances:

Water	1.00	Gold	19.25	Mercury	13.58
Alcohol	.79	Silver	10.47	Marble	2.7
Turpentine	.99	Lead	11.35	Cast Iron	7.2
Milk	1.03	Copper	8.95	Coal	1.3
Butter	.94	Tin	7.29	Cork	.24

1. A cubic foot of water weighs $62\frac{1}{2}$ lb., or 1000 oz. Find the weight of a cubic foot of copper.

2. Find the weight of a cubic foot of cast iron.

3. How much more does a cubic foot of gold weigh than a cubic foot of silver?

4. How much more does a cubic foot of copper weigh than a cubic foot of tin?

5. If a cubic foot of steel weighs 7830 oz., what is its specific gravity?

6. If a cubic foot of lead weighs 11,350 oz., what is its specific gravity?

7. How much more does a cubic foot of silver weigh than a cubic foot of tin?

SPECIAL PROBLEMS

LESSON 81

1. The sum of two numbers is 68, and $\frac{3}{4}$ of the smaller number equals $\frac{2}{3}$ of the larger number. What are the numbers?

SOLUTION. — If $\frac{3}{4}$ of the smaller number equals $\frac{2}{3}$ of the larger number, $\frac{1}{4}$ of the smaller number equals $\frac{1}{3}$ of $\frac{2}{3}$, or $\frac{2}{9}$ of the larger number, and $\frac{4}{4}$, or the smaller number, equals 4 times $\frac{2}{9}$, or $\frac{8}{9}$ of the larger number, which added to the larger number equals $\frac{17}{9}$ of the larger number, which equals 68. If $\frac{17}{9}$ of the larger number equals 68, $\frac{1}{9}$ of the larger number equals $\frac{1}{17}$ of 68, or 4, etc.

2. The sum of two numbers is 560, and $\frac{1}{3}$ of the smaller equals $\frac{1}{4}$ of the greater. What are the numbers?

3. If $\frac{1}{2}$ of John's money equals $\frac{3}{4}$ of Henry's, and they both have $960, how much money has each?

4. Divide 460 bu. of corn between A and B so that $\frac{3}{4}$ of A's share shall equal $\frac{2}{5}$ of B's share.

5. In a school there are 885 pupils. How many are there of each, if $\frac{3}{7}$ of the number of girls equals $\frac{5}{8}$ of the number of boys?

6. $\frac{2}{5}$ of A's land equals $\frac{2}{7}$ of B's, and they together have 432 acres. How many acres does each own?

7. If $\frac{3}{7}$ of A's money and $16 equals $\frac{8}{9}$ of B's money, and they both have $184, how much money has each?

8. If $\frac{4}{5}$ of B's land, less 10 acres, equals $\frac{8}{11}$ of C's land, and they both have 278 acres, how many acres has each?

9. In a child's savings bank there are 95 coins consisting of dimes and nickels. How much money is there in the bank if $\frac{5}{7}$ the number of nickels equals $\frac{5}{12}$ the number of dimes?

10. $83\frac{1}{3}\%$ of the difference between John's and William's fortune is \$1000. What is the fortune of each if $\frac{2}{5}$ of John's equals $\frac{1}{7}$ of William's fortune?

11. B and C can do $16\frac{2}{3}\%$ of a piece of work in a day, and 200% of what B can do equals what C can do. What part of the work can each do in a day?

12. 50% of the difference between two numbers is 240, and $12\frac{1}{2}\%$ of the first number is 25% of the second number. What are the numbers?

LESSON 82

1. A and B agreed to dig a ditch for \$72. A worked 8 da., and B 10 da. How much money should each receive?

SOLUTION. — Together they worked 10 da. plus 8 da., or 18 da. A's share should be $\frac{8}{18}$ of \$72, or \$32, and B's share should be $\frac{10}{18}$ of \$72, or \$40.

2. A and B built a house for \$14,000. A sent 6 men 50 da., and B 4 men 100 da. How much should each receive?

3. Two men agreed to build a wall for \$140. One man sent 6 men 7 da., and the other 7 men for 4 da. How much should each receive?

4. A and B agree to do a piece of work for \$260. A sends 10 men, and B 24 boys. How much should each receive if 3 boys can do as much work as two men?

5. Wilson and Brown rented a field of grass for \$11. Wilson pastured 8 horses for 3 weeks, and Brown 20 cows for 4 weeks. How much should each pay if 4 horses eat as much as 5 cows?

6. Two men, A and B, engaged to do a piece of work for $108. A sent 3 men 5 da., and B 6 boys 4 da. How much should each receive, if 1 man does as much as 2 boys?

7. M and N agree to excavate a street for $1300. M sends 10 men, and B 24 boys. How much should each receive if 3 boys can do as much as 2 men?

8. A lady purchased a certain number of yards of ribbon at 10¢ a yd. If she had paid 12¢ a yd. it would have cost her 56¢ more. How many yards did she buy?

SOLUTION. — Had she purchased at 12¢ a yd., she would have paid 2¢ a yd. more than by the first condition, and on all she would have paid 56¢ more. Hence she must have bought as many yards as 2¢ is contained times in 56¢, or 28. *Ans.* 28 yd.

9. A lady wishes to buy some silk. If she pays $2½ a yard she will lack $3 to pay for it, but if she pays $2 a yard, she will have money enough to pay for it. Find her money and the number of yards.

10. A man bought a number of sheep at $5 each, and had $25 left. If he had paid $7 each he would have had only $5 left. Find his money, and the number of sheep bought.

ALGEBRA

LESSON 83

1. Janson is 5 times as old as his son, but in 8 yr. he will be only 3 times as old. What is the age of each?

Let x = the son's age;
then $5x$ = Janson's age,
and $x + 8$ yr. = son's age in 8 yr.
and $5x + 8$ yr. = Janson's age in 8 yr.

By the condition of the problem,

$$5x + 8 \text{ yr.} = 3x + 24 \text{ yr.}$$
$$2x = 16 \text{ yr.}$$
$$\left.\begin{aligned} x &= 8 \text{ yr., the son's age,} \\ 5x &= 40 \text{ yr., Janson's age.} \end{aligned}\right\} \textit{Ans.}$$

2. A father is 4 times as old as his son, but in 20 yr. he will be twice as old. What is the age of each?

3. Thomas is $\frac{1}{5}$ as old as his father, but in 25 yr. he will be $\frac{3}{5}$ as old. Find the age of each.

4. The head of a fish is 6 in. long, the tail is as long as the head plus $\frac{1}{2}$ of the body, and the body is as long as the head and tail both. Find the length of the fish.

(1) Let x = length of the body.

(2) $\frac{x}{2} + 6$ in. = length of tail.

(3) $\frac{x}{2} + 12$ in. = length of head and tail,

which, by the condition of the problem, equals the length of the body.

(4) Hence $x = \frac{x}{2} + 12$ in.

(5) $2x = x + 24$ in.

(6) $x = 24$ in., length of body.

(7) $\frac{x}{2} + 6$ in. $= 18$ in., length of tail.

(8) 24 in. + 18 in. + 6 in. = 48 in., length of fish. *Ans.*

5. The head of a fish is 4 in. long, the tail is as long as the head plus $\frac{1}{3}$ of the body, and the body is as long as the head and tail both. Find the length of the fish.

6. A mother's age is twice her daughter's age, and 10 yr. ago her age was 3 times her daughter's age. Find the age of each.

7. Divide \$1000 among James, John, and Henry, so that James shall have \$72 more than John, and Henry \$100 more than James.

LESSON 84

1. Divide \$8800 among three persons, so that the first may receive twice as much as the second, and the second three times as much as the third.

2. Eight times a certain number minus 80, equals 5 times the number plus 10. Find the number.

3. What number is that to which if its $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ be added the result will equal 375?

4. A farmer raised 416 bu. of grain, consisting of wheat, corn, and rye. How many bushels were there of each, provided that the number of bushels of corn equaled three times the number of bushels of rye, and the number of bushels of rye equaled three times the number of bushels of wheat?

5. A boy has 64 pieces of money, consisting of cents, nickels, and dimes. The number of dimes is 6 times the number of nickels, and the number of cents is $\frac{1}{2}$ more than the number of dimes. How many *pieces* has he of each kind, and how *much* money has he?

6. Divide \$1022 among A, B, and C, so that B's share shall be $\frac{1}{3}$ of A's, and C's share $\frac{3}{4}$ of A's and B's together.

7. Wilson Smith is 4 times as old as his son; 5 years ago he was 7 times as old. Find their present ages.

8. A man, in buying some cloth, found that if he should pay \$3 a yard he would lack \$15 of having money enough to pay for it, but if he purchased cloth at \$2 a yard he would have \$25 left. How many yards did he buy?

9. A and B have 28 bu. of wheat between them. If A gives 3 bu. to B, he will then have 3 times as many bushels as B. How many bushels has each?

10. Divide \$4294 between A and B, so that 8 times A's share may equal 11 times B's share.

SUGGESTION. — Let $11x$ = A's, and $8x$ = B's.

11. At an election the number of votes cast for two candidates was 8712. How many votes did each receive if 5 times the number of votes cast for the successful candidate equaled 6 times the number cast for the unsuccessful candidate?

LESSON 85

1. James bought a certain number of lemons at 2¢ each, and as many oranges at 3¢ each, and paid in all 80¢. How many of each did he buy?

Let x denote the number of each bought,
then $2x$ = cost of lemons,
and $3x$ = cost of oranges,
and $5x = 80$ = cost of both,
$x = \frac{80}{5} = 16$, the number of each.

2. A butcher bought a certain number of sheep at \$5 each, and an equal number of calves at \$7 each. How many of each did he buy if he paid \$96 for all?

3. A tailor bought an equal number of yards of two different kinds of cloth for \$84. For one kind he paid \$4 a yard, and for the other kind \$3 a yard. How many yards of each kind did he buy?

4. A boy bought 4 lemons and 5 oranges for 28¢. What was the price paid for each if he paid twice as much for an orange as for a lemon?

Let x = the price of one lemon,
then $2x$ = the price of one orange,
and $4x$ = cost of the lemons,
and $10x$ = cost of the oranges.
Hence $14x = 28$¢, cost of lemons and oranges,
and $x = 2$¢, price of one lemon,
$2x = 4$¢, price of one orange. } *Ans.*

5. A man sold 16 apples and 6 oranges for 40¢. He paid 4 times as much for an orange as for an apple. Find the price of each.

6. A butcher bought 10 cows and 8 calves for \$240. He paid as much for 1 cow as for 4 calves. Find the price of each.

REVIEW WORK

LESSON 86

1. If 12 men and 12 boys can do a piece of work in 18 days, in what time can 18 men and 18 boys do the work, provided a boy does $\frac{2}{3}$ as much work as a man?

2. Divide \$280 between A and B in such ratio that $\frac{7}{8}$ of A's share plus \$5 equals B's share.

3. Amos can cut a field of grass in $\frac{3}{5}$ of a day, and Martin in $\frac{3}{8}$ of a day. In what time can both cut it?

4. Two ships sail from the same port. One goes due north 63 miles, and the other due west 84 miles. How far are the ships apart?

5. Express the temperature 30° C. in the Fahrenheit scale.

6. Find the corresponding Centigrade reading for 90° F.

7. Find when first after 1 o'clock the hands of a clock make an angle of 90°.

8. Find when first after 4 o'clock the hands make an angle of 90°.

9. The hour and minute hands are directly opposite at 6 o'clock. When will they be opposite again?

10. What is the difference in time between New York and a place 120° 40′ 15″ west of it?

11. Find the difference in longitude between two places if the difference in time is 4 hr. 10 min. 15 sec.

12. When it is 2.30 P.M. at Berlin, it is 58 min. 41 sec. past 7 A.M. at Cincinnati. What is the longitude of Cincinnati, the longitude of Berlin being 13° 23′ 45″ east?

13. Find the square root of 811,801; 16,892,100; $\frac{36}{625}$.

14. Divide 468 into two parts, that $\frac{1}{5}$ of one part shall equal $\frac{2}{3}$ of the other.

LESSON 87

The difference between the weight of a body in air and its weight in water is equal to the weight of water displaced. Therefore the specific gravity of a solid heavier than water equals its weight in air divided by its loss of weight in water. (See Lesson 80.)

1. A solid weighing 695 oz. in air loses 83 oz. when weighed in water. Find its specific gravity.

2. If a piece of brass weighs 2100 lb. in air and 1850 lb. in water, what is its specific gravity?

3. A piece of coal weighs 185.5 lb. in air and 115.5 lb. in water. Find its specific gravity.

4. A parlor 6 yd. wide requires 64 yd. of Brussels carpet. What is the length of the parlor?

5. The diagonal of a trapezium is 363 ft. The perpendiculars to the diagonal are 90 ft. and 108 ft. respectively. What is its area?

6. Compare the perimeter of a rectangle 80 rd. long and 45 rd. wide with the perimeter of a square of equal area.

7. How many square yards of canvas will be required to make a conical tent 12 ft. in diameter and 18 ft. high?

AVERAGE TERM OF CREDIT

LESSON 88

The **Average Term of Credit** is the time to elapse before several sums of money, without interest, due at different times, may be paid at once, without loss to the parties concerned.

1. A owes B $500 due in 4 mo., $800 due in 5 mo., $1400 due in 6 mo. Find the average term of credit.

A credit of $500 for 4 mo. = a credit of $1 for 500 times 4 mo., or 2000 mo. A credit of $800 for 5 mo. = a credit of $1 for 800 times 5 mo., or 4000 mo. A credit of $1400 for 6 mo. = a credit of $1 for 8400 mo.

By adding we find $1 has a credit of 14,400 mo. If $1 has a credit of 14,400 mo., $2700 will have a credit of $\frac{1}{2700}$ of 14,400 mo., or $5\frac{1}{3}$ mo., the average term of credit.

OPERATION

$500 \times 4 = 2000$
$800 \times 5 = 4000$
$1400 \times 6 = 8400$
$2700 \overline{)14400}$
$5\frac{1}{3}$

$5\frac{1}{3}$ mo. *Ans.*

2. I owe $500 due in 6 mo., $600 in 8 mo., and $900 in 4 mo. What is the average term of credit?

3. A man owes $600 due in 4 mo., $800 due in 5 mo., and $1400 due in 6 mo. What is the average term of credit?

4. A merchant bought a bill of goods amounting to $2250, of which $1250 was to be paid in 30 da., $750 in 60 da., and the remainder in 90 da. Find the average term of credit.

5. C owes D $8000, which he agrees to pay as follows: $3000 cash, $2000 in 4 mo., and the balance in 8 mo. Find the average term of credit.

OPERATION

3000 cash . . . no credit
2000 × 4 = 8000
3000 × 8 = 24000
8000)32000
4 . . . 4 mo. *Ans.*

6. Johnson owes Smith $1800, of which he agrees to pay $800 down, $600 in 4 mo., and $400 in 9 mo. Find the average term of credit.

7. If Morgan lends his brother $1350 for 6 mo., how long ought his brother to lend him $900 to balance the favor?

The use of $1350 for 6 mo. is equivalent to the use of $1 for 1350 times 6 mo., or 8100 mo., or to the use of $900 for $\frac{1}{900}$ of 8100 mo., or 9 mo.

OPERATION

1350 × 6 = 8100
8100 ÷ 900 = 9
9 mo. *Ans.*

8. If I borrow $150 from a friend for 3 mo., how many months should I lend him $300 to repay the favor?

9. I owe $1100 to be paid in 8 mo. without interest. If I pay $300 at the end of 4 mo., and $400 at the end of 6 mo., how long after maturity may I retain the remainder as an equivalent for the sum prepaid?

Since the first payment ($300) is made 4 mo. before it is due, I am entitled to a credit of $300 for 4 mo., or of $1 for 1200 mo. Likewise on the second payment I am entitled to a credit of $1 for 800 mo. On both I am entitled to a credit of $1 for 2000 mo. Hence to balance this credit, I may retain the remainder ($400) after maturity for $\frac{1}{2000}$ of 2000 mo., or 5 mo.

OPERATION

300 × 4 = 1200
400 × 2 = 800
700 2000
1100 − 700 = 400
2000 ÷ 400 = 5
5 mo. *Ans.*

10. C owes D $2400 payable in 8 mo. without interest. At the end of 2 mo. he pays $600, at the end of 3 mo., $800, and at the end of 4 mo., $200. How long in equity may C retain the remainder after maturity?

LESSON 89

The **Equated Time** is the date at which several sums due at different times may be paid at once, without loss to the parties concerned.

1. A man bought goods April 1, 1902, as follows: $700 on a credit of 2 mo., $1000 on a credit of 3 mo., $1400 on a credit of 6 mo. What is the equated time of payment?

We find the average term of credit to be $4\frac{4}{31}$ mo. Hence the equated time is $4\frac{4}{31}$ mo. from April 1, or Aug. 5.

OPERATION

$700 \times 2 = 1400$
$1000 \times 3 = 3000$
$1400 \times 6 = 8400$
$3100 \quad)12800$
$4\frac{4}{31}$

April 1 + $4\frac{4}{31}$ mo. = Aug. 5. *Ans.*

2. Mr. Day bought merchandise Jan. 1, 1902, as follows: $1050 on 2 mo., $700 on 4 mo., and $350 on 6 mo. What is the equated time of payment?

3. Find the equated time for the payment of $3000 from June 11, 1902, due as follows: $\frac{1}{5}$ in 30 da., $\frac{1}{4}$ in 60 da., $\frac{3}{10}$ in 90 da., and the remainder in 120 da.

4. I bought goods Aug. 8, 1902, to the amount of $4200 to be paid as follows: $2100 cash, $1400 in 4 mo., and the remainder in 6 mo. What is the equated time of payment?

5. I borrowed $720 for 10 mo., and $1080 for 8 mo. At the end of 7 mo. I paid $1200. How long after the equated time for the payment of the whole may I retain the remainder?

6. A man owed $5000 to be paid in 1 yr. 3 mo. At the end of 5 mo. he paid $1600, and 2 mo. later he paid $1800 more. When in equity should he pay the balance?

ANNUAL INTEREST

LESSON 90

Annual Interest is simple interest computed on the principal, and on each year's unpaid interest from the time it is due until settlement.

NOTE. — Annual interest is recognized by some states as reasonable and just when the obligation contains the words "with interest payable annually."

1. Find the amount due on a note for $800, at 6%, for 3 yr. 8 mo. 21 da., with interest payable annually, if no interest was paid until settlement.

OPERATION

Annual interest = $800 × .06 = $48.

Amt. of $800, at 6%, for 3 yr. 8 mo. 21 da.,	$978.80
Int. of $48, at 6%, for 2 yr. 8 mo. 21 da.,	7.848
Int. of $48, at 6%, for 1 yr. 8 mo. 21 da.,	4.968
Int. of $48, at 6%, for 8 mo. 21 da.,	2.088
Amt. due at settlement,	$993.704 *Ans.*

The above operation may be shortened by finding the interest of $48 for the sum of these periods:

yr.	mo.	da.	
2	8	21	
1	8	21	
	8	21	
5	2	3	= aggregated time.

Amt. of $800, at 6%, for 3 yr. 8 mo. 21 da.,	$978.80
Int. of $48, at 6%, for 5 yr. 2 mo. 3 da.,	14.904
Amt. due at settlement,	$993.704 *Ans.*

STATEMENT

44.7 mo. = time prin. bears int.
62.1 mo. = aggregated time of unpaid int.

$(\$800 \times .06) \div 12 \times 44.7 + \$800 + (\$48 \times .06) \div 12 \times 62.1 =$ answer.

2. Find the amount due on a note for $3000, at 6%, annual interest, for 3 yr. 6 mo., if no interest has been paid.

3. At 6%, annual interest, find the amount due on a note for $3650, if it ran 2 yr. 6 mo. 6 da., and no interest was paid till settlement.

4. Find the amount due on a note for $600, dated March 4, 1898, with annual interest, at 6%, if it was paid July 20, 1902, no interest being paid till settlement.

5. Find the amount due April 20, 1902, on a note for $860, at 5%, annual interest, dated Nov. 27, 1898, if the interest was paid for the first two years.

6. A note for $2460, dated Jan. 8, 1898, at 5% annual interest, was paid Jan. 16, 1902. Find the amount due, the interest being regularly paid for 3 years.

7. What is the difference between the annual interest and the compound interest of $8500 for 3 yr. 6 mo. at 6%?

FOREIGN EXCHANGE

LESSON 91

Foreign Exchange is the exchange between places in different countries.

A **Bill of Exchange** is a written order upon one party to pay a certain sum of money to another party, or to his order, at a specified time.

NOTE. — Most foreign bills of exchange are drawn upon the leading banking houses of the chief commercial centers, as London, Paris, Berlin, Bremen, etc., and are expressed in the currency of the corresponding country.

Bills of exchange drawn upon London are called **Sterling Bills.** The legal par of exchange between the United States and Great Britain is $4.8665 per *pound sterling.*

Bills of exchange drawn upon Paris are reckoned at a certain number of *francs* to the dollar, the value of the franc being about $.193.

Bills of exchange drawn upon Berlin, Hamburg, and Bremen are reckoned at a certain number of cents per four reichsmarks (marks), the value of the reichsmark being about $.238.

NOTE. — The above rates of exchange are not fixed values. Rates of exchange vary with the conditions of trade. They are quoted by the commercial papers.

Foreign bills of exchange are usually issued in sets of two or three bills, called the first, second, and third of ex-

change. In order to provide against loss or delay, these bills are sent, each by a different route, or by a different means of conveyance. Each bill contains a condition that when one has been paid the others become void.

1. What is the cost of a bill of exchange on London for £ 130, when exchange is at $ 4.855?

Since the value of £ 1 is $ 4.855, the value of £ 130 is 130 times $ 4.855, or $ 631.15.

OPERATION

$ 4.855
130
145650
4855
$ 631.150 *Ans.*

2. Find the cost in New York of a bill of exchange on London for £ 375 10 *s.* when exchange is at $ 4 875.

3. Find the cost of a bill of exchange on Liverpool for £ 475 16 *s.* 9 *d.* when exchange is at $ 4.81½.

4. Find the cost of a bill of exchange on Paris for 3090 francs, exchange being 5.15 francs to the dollar.

Since 5.15 francs cost $1, the cost of 3090 francs will be as many dollars as 5.15 is contained times in 3090, which equals $ 600.

OPERATION

5.15)3090($ 600. *Ans.*
3090
00

5. What will be the cost in Baltimore of a bill of exchange on Paris for 12,315 francs, when exchange is quoted at 5.3 francs to the dollar?

6. What will be the cost in Boston of a bill of exchange on Berlin for 5520 marks, exchange being quoted at 96½?

Since the cost of 4 marks is $ .96½, the cost of 1 mark is ¼ of $ .96½, or $ .24⅛, and the cost of 5520 marks is 5520 times $ .24⅛, or $ 1331.70.

OPERATION

$ .96½ ÷ 4 = $ .24⅛ = cost of 1 mark.
$ .24⅛ × 5520 = $ 1331.70. *Ans.*

7. Find the cost in Philadelphia of a bill of exchange on Bremen for 3680 marks, exchange being $94\frac{3}{4}$.

8. Find the cost of a bill of exchange on Dublin for £ 1550 15 *s.*, sterling exchange being $ 4.845.

9. Find the cost of a bill of exchange on Frankfort for 11,040 marks, when exchange is quoted at $96\frac{1}{2}$.

NOTE. — The principles involved in the preceding examples are applicable to exchange with any other nation. By consulting the table of foreign money and applying the methods illustrated in the preceding examples, any problem in foreign exchange may be easily solved.

TABLES

LONG MEASURE

Long Measure is used in measuring lines or estimating distances.

The units employed in measuring length are, the *inch*, the *foot*, the *yard*, the *rod*, the *mile*, the *link*, and the *chain*.

LONG MEASURE	SURVEYORS' LONG MEASURE
12 in. = 1 ft.	7.92 in. = 1 li.
3 ft. = 1 yd.	25 li. = 1 rd.
$5\frac{1}{2}$ yd. / $16\frac{1}{2}$ ft. = 1 rd.	4 rd. / 66 ft. = 1 ch.
320 rd. = 1 mi.	80 ch. = 1 mi.

1 mi. = 1760 yd. = 5280 ft. = 63,360 in.

OTHER MEASURES. — 12 lines = 1 inch; 3 barleycorns = 1 inch; 4 inches = 1 hand, used in measuring the height of horses; 3.3 feet = 1 pace, used in approximating distances; 6 feet = 1 fathom, used in measuring depths at sea; 120 fathoms = 1 cable length; 6086.7 feet = 1 knot, a nautical or geographical mile; 60 geographical, or 69.16 common miles = 1 degree of latitude or longitude at the equator; 40 rods = 1 furlong; 8 furlongs = 1 mile; 3 miles = 1 league.

The Surveyors' chain = 4 rods = 66 feet = 792 inches long. The Engineers' chain is 100 feet long, divided into 100 links of 1 foot each.

SQUARE MEASURE

Square Measure is used in estimating areas or surfaces, as land, boards, etc.

The units employed are: the *square inch;* the *square*

foot; the *square yard;* the *square rod;* the *acre;* the *township;* the *square mile;* the *square link,* and the *square chain.*

SQUARE MEASURE	SURVEYORS' SQUARE MEASURE
144 sq. in. = 1 sq. ft.	625 sq. li. = 1 sq. rd. or perch.
9 sq. ft. = 1 sq. yd.	16 sq. rd. = 1 sq. ch.
$30\frac{1}{4}$ sq. yd. } = 1 sq. rd.	10 sq. ch. = 1 A.
$272\frac{1}{4}$ sq. ft. } or perch.	640 A. = 1 sq. mi.
160 sq. rd. or perches = 1 A.	36 sq. mi. = 1 Tp.

CUBIC, OR SOLID, MEASURE

Cubic, or **Solid, Measure** is used in estimating all things that have the dimensions length, breadth, and thickness.

The principal units employed are the *cubic inch;* the *cubic foot;* the *cubic yard.* A cubic yard of earth is called a *load.*

TABLE

1728 cu. in. = 1 cu. ft.
27 cu. ft. = 1 cu. yd.
16 cu. ft. = 1 cd. ft.
8 cd. ft. } = 1 cd.
128 cu. ft. }

A *perch* of stone or of masonry is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high. It contains $24\frac{3}{4}$ cu. ft.

A *cord* of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high. The *cord foot,* which is practically obsolete, is $\frac{1}{8}$ of a cord, or 16 cu. ft.

MEASURES OF CAPACITY

Measures of **Capacity** are used in estimating quantity of fluids and many dry substances.

There are two sets of measures of capacity, one for liquids and one for dry substances.

The standard unit of **Liquid Measure** is the *gallon*, which contains 231 cu. in. The standard unit of **Dry Measure** is the *bushel*. It contains 2150.42 cu. in. Dry measure is used in measuring grain, fruit, lime, etc.

LIQUID MEASURE	DRY MEASURE
4 gi. = 1 pt.	2 pt. = 1 qt.
2 pt. = 1 qt.	8 qt. = 1 pk.
4 qt. = 1 gal.	4 pk. = 1 bu.

A pint of water weighs about a pound. A cubic foot of distilled water weighs about 1000 oz., or 62½ lb.

The *barrel* and *hogshead* do not express fixed quantities; they vary in different states. In estimating the capacity of cisterns, reservoirs, etc., the barrel is generally considered to contain 31½ gal., and the hogshead 63 gal.

MEASURES OF WEIGHT

The **Weight** of a body is the measure of the earth's attraction for it.

There are four sets of measures of weight: *Troy; Avoirdupois; Apothecaries'*; and *Apothecaries' Fluid.*

Troy Weight is used in weighing jewels and the precious metals, as gold and silver. The unit of weight is the *pound*, which contains 5760 gr.

Avoirdupois Weight is used in weighing nearly everything except gold, silver, and jewels. The unit of weight is the *pound.* It contains 7000 Troy grains.

The *carat* is a unit of 4 imaginary grains employed in rating diamonds and precious stones, as the ruby, topaz, emerald, etc. The

term is also used to express the fineness of gold. Thus, gold 18 carats fine consists of 18 parts pure gold, alloyed with 6 parts of some other metal, the whole mass being divided into 24 equal parts.

TROY WEIGHT	AVOIRDUPOIS WEIGHT
24 gr. = 1 pwt.	16 oz. = 1 lb.
20 pwt. = 1 oz.	100 lb. = 1 cwt.
12 oz. = 1 lb.	20 cwt. = 1 T.

At the United States customhouses, in invoices of imported goods, and in the wholesale trade of iron and coal, the ton of 2240 lb. is generally used.

The following table will show the number of avoirdupois pounds in a bushel of the principal farm products, as fixed by law:

COMMODITIES	WEIGHTS	COMMODITIES	WEIGHTS
Barley	48 lb.	Oats	32 lb.
Beans	60 lb.	Potatoes	60 lb.
Clover seed	60 lb.	Rye	56 lb.
Corn in the ear	70 lb.	Timothy seed	45 lb.
Corn shelled	56 lb.	Wheat	60 lb.

Some other denominations in common use:

56 lb. of butter = 1 firkin.	100 lb. dry fish = 1 quintal.
84 lb. of butter = 1 tub.	196 lb. flour = 1 barrel.
100 lb. of nails = 1 keg.	200 lb. pork or beef = 1 barrel.

APOTHECARIES' WEIGHT

Apothecaries' Weight is used by physicians and apothecaries in prescribing and preparing dry medicines. The unit is the *pound*, which contains 5760 gr., like the Troy pound. Medicines are bought and sold by avoirdupois weight.

Apothecaries' Fluid Measure is used by physicians and apothecaries in prescribing and preparing liquid medicines.

APOTHECARIES'

WEIGHT	APOTHECARIES' FLUID MEASURE
20 gr. = 1 ℈.	60 minims (♏) = 1 fluid dram (f ʒ).
3 ℈ = 1 ʒ.	60 fluid drams = 1 fluid ounce (f ℥).
8 ʒ = 1 ℥.	16 fluid ounces = 1 pint (O.) (Octavus).
12 ℥ = 1 lb.	8 pints = 1 gallon (Cong.) (Congius).

TIME

The unit of **Time** is the *day*.

TABLE

60 sec. = 1 min.
60 min. = 1 hr.
24 hr. = 1 da.
7 da. = 1 wk.
365 da. = 1 common year.
366 da. = 1 leap year.
100 yr. = 1 cen.

The year is divided into 12 periods, called calendar months. The following table will show the names of the calendar months and the number of days in each:

TABLE

NAMES OF MONTHS	DAYS IN EACH	NAMES OF MONTHS	DAYS IN EACH
1. January	31	7. July	31
2. February	28 or 29	8. August	31
3. March	31	9. September	30
4. April	30	10. October	31
5. May	31	11. November	30
6. June	30	12. December	31

Four weeks constitute what is called a *lunar month;* 13 lunar months and 1 day make a common year, 365 da.

The year is also divided into periods of 3 months, each constituting what is called a season. December, January, and February form the winter season; March, April, and May, spring; June, July, and August, summer; and September, October, and November, fall, or autumn.

By committing to memory the following stanza, the number of days in each month may be readily remembered:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except February alone,
Which has but twenty-eight in fine,
Till leap year gives it twenty-nine."

NOTE. — In business transactions it is customary to consider 30 days as a month, and 12 months a year.

UNITED STATES MONEY	ENGLISH MONEY
10 mills = 1 ¢.	4 far. = 1 *d.*
10 ¢ = 1 d.	12 *d.* = 1 *s.*
10 d. = $1.	20 *s.* = 1 £.
$10 = 1 E.	21 *s.* = 1 guinea.

CIRCULAR MEASURE

TABLE

60 seconds (″) = 1 minute (′).
60 minutes = 1 degree (°).
360 degrees = 1 circumference (C.).

The term **Sign** is sometimes used to express 30°, from the fact that the ancients divided the zodiac into 12 parts

of 30° each, and represented each part by an arbitrary sign.

To distinguish these from minutes and seconds of *time*, the phrase *of arc* is employed. Thus, 30″ is read, 30 seconds of arc.

COUNTING	PAPER
12 units = 1 doz.	24 sheets = 1 quire (qr.).
12 doz. = 1 gro.	20 quires (480 sheets) = 1 ream (rm.).
12 gro. = 1 grt. gro.	2 reams = 1 bundle (bun.).
20 units = 1 score.	5 bundles = 1 bale (B.).

ANSWERS

Page 6. — **14.** $\frac{81}{480}$. **Lesson 2.** — **1.** $66.96. **3.** 1 yd. **4.** $28\frac{1}{2}$. **5.** $39\frac{1}{10}$ acres, C's; $117\frac{3}{10}$ acres, all. **6.** 55.708 in. perimeter; 157.08 sq. in. area.

Page 7. — **7.** $3456. **8.** 1432. **9.** 321 A. 40 P. **10.** $21.85. **11.** $463.83\frac{1}{3}$. **12.** $180. **13.** 23.562 sq. ft. **14.** 25.1328 cu. ft. **Lesson 3.** — **1.** $54\frac{22}{37}\%$.

Page 8. — **3.** $\frac{22}{4}$ ft.; $\frac{1}{66}$ rd.; $\frac{25}{33}\%$. **4.** $12\frac{1}{2}\%$. **5.** 432 bbl. **6.** $27\frac{8}{11}$ rd. **7.** $2.88\frac{3}{4}$. **8.** $144. **9.** .0012. **10.** $281\frac{1}{4}$. **11.** $12\frac{1}{2}\%$; $1\frac{9}{16}\%$. **12.** $\frac{5}{6}$. **13.** 44. **14.** 21 yr. **15.** $37\frac{1}{2}\%$. **16.** $\frac{1}{4}$. **17.** $\frac{9}{10}$. **18.** $3861\frac{1}{3}$ bu.

Page 9. — **1.** $\frac{3}{5}$. **2.** $1.54. **3.** $40.866. **4.** 35 bu. **5.** $33\frac{1}{3}\%$ males; $66\frac{2}{3}$ females. **6.** $33\frac{1}{3}\%$ water; $66\frac{2}{3}\%$ milk. **7.** 1. **8.** $\frac{7}{9}$. **9.** 3200 bd. ft. **10.** 50%. **11.** 36.16+ bu.

Page 10. — **12.** $117,000. **13.** $3835.117+. **Lesson 5.** — **1.** 33 bu.; 12 bu. **2.** $66\frac{2}{3}$ cu. ft. **3.** 201.0624 sq. ft. **4.** 50 hr. **5.** 6 ft. **6.** $3017.20. **7.** 11 min. 56 sec. past 3 P.M. **8.** 84° 24′ west. **9.** 1650.

Page 11. — **10.** $9774.968. **11.** $200, loss. **12.** 25 bd. ft. **13.** $112. **Lesson 6.** — **1.** 128 sq. in.; it is half as large. **2.** 94 rd. **3.** About .803+. **4.** 5 min. 11 sec. past 5 P.M. **5.** $2003\frac{1}{13}$; $2670\frac{10}{13}$; $4006\frac{2}{13}$. **6.** 108 sq. ft.

Page 12. — **9.** 105 bu. **10.** $142\frac{1}{2}$. **11.** 28800. **12.** $9.21. **13.** Lose 9%.

Page 14. — **2.** $375, A; $300, B. **3.** $90, A; $115, B; $148, C. **4.** $3825, A; $4050, B; $5400, C. **5.** $7500, A; $6500, B; $8400, C.

Page 15. — **2.** $1368, A; $760, B; $342, C. **3.** $1080, Smith; $600, Jones. **4.** $750, Samson; $300, Straw; $200, Riddle. **5.** $280, Wilson; $350, Allen; $420, Thompson. **6.** $5670, Anson; $7560, Roll. **7.** $624, Payne; $676, Collins; 14 mo., Arnold.

Page 18. — **2.** $106.968. **3.** $223.104. **4.** $5568.526. **5.** $5277.507. **6.** $111.022. **7.** $54.68.

Page 21. — **8.** $2400. **9.** $1376.14. **10.** $210.204. **11.** $41.80. **12.** $3.374 int. more.

Page 22. — **13.** At $4\frac{1}{2}$ on 4 mo. is 44¢ more profitable. **14.** $54.059. **15.** $30.297. **16.** $5333.094.

Page 23. — **2.** $1600, A; $2400, B; $3200, C. **3.** $400; $600. **6.** 180 sq. ft. **7.** 733.344 cu. in. **8.** 1 yr. 6 mo. **9.** $10253.164+. **10.** $46.75.

Page 24. — **11.** $63, A; $105, B; $147, C. **12.** $494. **Lesson 12.** — **1.** $612.425. **2.** $120334\frac{1}{2}$ sq. ft. **3.** 40 sq. rd. **4.** $12,800. **5.** 11 A.M. **6.** 1400. **7.** $3. **8.** $49.152.

Page 25. — **9.** $2.52; 20%. **10.** $168, A; $201.60, B; $192, C. **11.** 216. **12.** 48 yr. **13.** $15940000. **14.** 6249.96 mi. **15.** 186000 mi.

Page 28. — **1.** $7490.06. **2.** $5241.91. **3.** $10974.20.

Page 29. — **4.** $7011.74. **6.** $111.68. **7.** $6215.08.

Page 33. — **2.** $4663\frac{1}{3}$. **3.** $2453\frac{1}{3}$.

Page 34. — **7.** 4%. **8.** $320; $6\frac{2}{13}$%. **Lesson 18.** — **1.** 72 shares. **2.** 120 shares.

Page 35. — **3.** 79 shares; $31 surplus. **5.** $9000. **6.** $560000; 500 shares.

Page 36. — **9.** $522.50. **10.** $382.50.

Page 38. — **3.** $5250. **4.** $1900. **Lesson 20.** — **1.** $450. **2.** $5281.25. **3.** 75. **4.** 40. **6.** $36000.

Page 39. — **8.** $36630. **Lesson 21.** — **2.** $5\frac{5}{14}$%. **3.** $6\frac{2}{13}$%. **4.** $7\frac{1}{7}$%. **5.** $5\frac{5}{9}$%.

Page 40. — **7.** $80. **8.** $133\frac{1}{3}$. **9.** $125. **11.** 200 shares. **12.** $30000; 300 shares. **Lesson 22.** — **1.** $250. **2.** $180. **3.** $28355.

Page 41. — **4.** $7\frac{1}{2}$%. **5.** $4\frac{1}{2}$%. **6.** 5%; 6%. **7.** Increased $5; 133 shares; $30 surplus. **8.** No difference. **9.** $7\frac{1}{2}$%; $8\frac{8}{9}$%. **10.** $80.

Page 42. — **1.** The circumference of the circle is 2.2656 ft. greater. **2.** The volume of the prism is 3 times that of the pyramid. **3.** 50 da. **4.** 20%. **6.** $18\frac{1}{2}$ ch. **7.** 4 ch. **8.** 960 bd. ft. **9.** $16\frac{3}{4}$. **10.** 33 bd. ft.

Page 43. — **11.** 2412.7488 cu. in. **12.** 1908.958+ lb. **13.** $739\frac{1}{8}$ times. **Lesson 24.** — **1.** 432 bu. **2.** $750. **3.** 4. **6.** $505.689. **7.** $2409.243. **8.** $16000. **9.** $9010.

Page 44. — **12.** $17\frac{3}{16}$. **13.** Feb. 20, 1910. **14.** $435. **15.** $21. **16.** 8%. **Lesson 25.** — **6.** 3391. **7.** $19.60.

Page 45. — **8.** $416\frac{2}{3}$. **9.** 45; 51. **10.** $32.17. **11.** $810.

Page 48. — **2.** Willis, 24 bu.; John, 12 bu. **4.** 60¢, R.; 20¢, E. **5.** $200, horse; $50, carriage. **6.** 90; 18.

Page 49. — **2.** \$3, hat; \$6, shoes; \$24, coat. **3.** \$4222⅔, 1st; \$12666⅔, 2d; \$21111⅓, 3d. **4.** 9. **5.** 9. **7.** 24; 96. **8.** 128 bu. oats; 512 bu. wheat. **9.** \$340; \$85.

Page 50. — **1.** 24. **2.** 36. **3.** 28. **4.** 23. **5.** 7. **6.** 27. **7.** 24. **8.** 10. **9.** 21. **10.** 12. **11.** 8. **12.** 32. **13.** 30. **14.** 14. **15.** 8. **16.** 10. **17.** 12; 36; 72. **18.** 8; 24; 64.

Page 51. — **19.** 8 qt.; 16 qt.; 48 qt. **20.** 17; 51. **21.** 32 rd.; 64 rd. **22.** 9 bu. rye; 18 bu. corn; 36 bu. oats.

Page 52. — **2.** \$300. **3.** 180; 270. **4.** 12 yr. **5.** \$6000. **7.** 480.

Page 53. — **4.** 20. **5.** 24. **6.** 30. **7.** 36. **8.** 44. **9.** 45. **10.** 21. **11.** 15. **12.** 30. **16.** 30. **17.** 36. **18.** 60. **19.** 96. **20.** 72. **21.** 400. **22.** 100. **23.** 60. **24.** 36.

Page 54. — **4.** 16 yr. **5.** \$10, hat; \$16, coat.

Page 55. — **6.** 35 mi. **7.** 10, Mary; 6, sister. **8.** 60. **9.** \$20, M.; \$32, E. **10.** \$140. **11.** 29. **12.** 52. **13.** 47. **14.** 39. **15.** 106. **16.** 85. **17.** 44. **18.** 80. **19.** 105. **20.** 120. **21.** 24. **22.** 22. **23.** 35. **24.** 82. **25.** 42. **26.** 24.

Page 56. — **1.** 16. **2.** 6. **3.** 77. **4.** 39. **5.** 202. **6.** 3. **7.** 29. **8.** 4. **9.** 9. **11.** 5. **12.** 7. **13.** 3. **14.** 4. **15.** 1. **16.** 8. **17.** 2. **18.** 4.

Page 58. — **4.** \$80. **6.** 8. **7.** \$376; \$532; \$782. **8.** \$8000.

Page 59. — **1.** \$28, R.; \$84, W. **2.** 70; 280. **3.** 60 mi. **4.** 8; 24; 48. **5.** 28 yr. **6.** 12 yr.; 48 yr. **7.** 36. **8.** 48¢. **9.** \$96, H.; \$72, J.; \$48, W. **10.** 24 in. **11.** 7; 18.

Page 61. — **10.** 24 yr., sister; 18 yr., John. **11.** 7 qt., W.; 35 qt., H.; 21 qt., J. **12.** \$600, sheep; \$1200, cows; \$1800, horses.

Lesson 37. — **1.** 36 da. **2.** 40 da. **3.** \$7.776. **4.** $5\frac{5}{19}\%$. **5.** \$84. **7.** \$250,000. **8.** \$27, A; \$36, B; \$39, C.

Page 62. — **10.** 672 sq. in. **13.** 402.1248 cu. in. **14.** 15 ft. **15.** 2%. **16.** \$323.232. **17.** \$3366.80.

Page 64. — **1.** 64; 729; 169; 400; 625; 1728; $2\frac{1}{4}$; .81; 12.25; $49\frac{8}{27}$; $447\frac{424}{625}$. **2.** 16807; 32,768. **3.** 400; 4096; 144000. **4.** $75\frac{1}{9}$; $29\frac{52}{81}$; $277\frac{7}{9}$. **5.** 1.898; $53\frac{7}{9}$. **6.** 41.64+; $\frac{343}{3375}$. **7.** .000000216; 903.904225. **8.** 7^7. **9.** 9^{14}. **10.** $(\frac{2}{3})^{14}$. **11.** 4th power. **12.** 6th power. **13.** 6th power. **14.** 16. **15.** 32. **16.** $\frac{1}{2}$. **17.** $\frac{1}{3}$. **18.** 5.

Page 69. — **4.** 499. **5.** 563. **6.** 591. **7.** 696. **8.** 758.

Page 72. — **1.** 14. **2.** 19. **3.** 24. **4.** 46. **5.** 79. **6.** 216. **7.** 307. **8.** 435. **9.** 726. **10.** 806. **11.** .24. **12.** .35. **13.** .08. **14.** .05108. **15.** .413. **16.** 7.3. **17.** 34.0917. **18.** 86.01. **19.** $\frac{8}{9}$. **20.** $\frac{14}{15}$. **21.** $\frac{14}{15}$.

22. $\frac{21}{25}$. **23.** $\frac{7}{8}$. **24.** .6546+. **25.** 2.738. **26.** 3.093+. **27.** 2.217+. **28.** $8\frac{3}{4}$. **29.** 3.824+. **30.** $\frac{2}{13}$. **31.** 3.561. **32.** 5.0074. **33.** .02108.

Page 73. — **1.** 12 ft. **2.** 80 rd. **3.** 16.5 ft. **4.** 742. **5.** $96. **6.** 220 yd. **7.** 6 sq. ft. **8.** $1.80. **9.** 80 rd. long; 20 rd. wide. **10.** 90 ft. long; 40 ft. wide.

Page 81. — **1.** 14. **2.** 25. **3.** 56. **4.** 123. **5.** 336. **6.** 607. **7.** 705. **8.** 6.38. **9.** 2.34. **10.** 25.46. **11.** 2.98+. **12.** .6. **13.** .07. **14.** $\frac{3}{7}$. **15.** $\frac{8}{13}$. **16.** .4641. **17.** 1.81712. **18.** .854+; $2\frac{2}{3}$; $16\frac{3}{4}$; .961+; .529+. **19.** 24. **20.** $20\frac{1}{4}$ sq. ft. **21.** 4.63 ft. **22.** 337.5 sq. ft.

Page 82. — **4.** $820.869, P. W.; $123.13, Dis. **6.** 27. **7.** 6.2832 ft.

Page 83. — **8.** .8584 sq. ft. **9.** 1152. **11.** 1.40\frac{1}{4}$. **Lesson 51.** — **1.** 95%. **2.** $32. **3.** $60 per share. **4.** $1680, A; $2016, B; $5544, C. **5.** $26580. **6.** 782.258 sq. ft. **7.** 41.2032 cu. in. **8.** $693, A; $660, B; $726, C.

Page 84. — **9.** $117\frac{3}{20}$ A. **10.** 159.068 mi. **Lesson 52.** — **1.** $765, A; $910, B; $825, C. **2.** $56\frac{2}{3}$; $23\frac{1}{3}$. **4.** The second has a volume of 8 times the first. **5.** $272\frac{1}{2}$ ft. **6.** 13.416+ rd. **7.** 3 hr. 37 min. 30 sec. **8.** 162 ft. **9.** 15 in. **10.** 20 lb. **11.** $333\frac{1}{3}$ lb.

Page 85. — **1.** $46750. **2.** $2.75. **3.** 90% gold; 5% silver; 5% copper. **4.** 7 hr. 2 min. 41 sec.; Boston. **5.** 8.81+ ft. **6.** 120. **7.** $42; $18. **8.** 79 A. 2 sq. rd. **9.** 23 A. 20 sq. rd. **10.** 30. **11.** $120.

Page 87. — **6.** 504 sq. rd. **7.** 840 sq. yd. **8.** 205.8 sq. ch. **9.** 80 rd.

Page 88. — **10.** 55 yd. **11.** 150 sq. ft. **12.** $5100. **13.** 780.62 sq. rd. **14.** $19.20. **15.** They are equal.

Page 89. — **4.** 120 mi.

Page 90. — **5.** 30 ft. **6.** 114 ft. **7.** 33 ft. **8.** 271.75 ft. **9.** 106.12 ft. **10.** 43.86 ft.

Page 91. — **6.** 750 sq. rd. **7.** 700 sq. yd. **8.** $18\frac{3}{4}$ A. **9.** 40 sq. ft. **10.** 2100 sq. ft. **11.** 517.42 sq. rd. **12.** 2592 sq. ft. **13.** 960 sq. ft. **14.** 972 sq. yd.

Page 92. — **7.** 94.248 rd. **8.** 3 rd. **9.** 30 rd. **10.** 94.248 rd. **11.** $12\frac{1}{2}$ A. **12.** 110.85 sq. rd.

Page 93. — **1.** 14.14+ ft. **2.** 22.62+ yd. **3.** 32 sq. ft. **4.** 7.07+ ft. **5.** 231.588+ sq. ft. **6.** 35 ft.

Page 95. — **10.** 2700 sq. ft. **11.** 207 sq. ft. **12.** 2880 sq. ft.; about 230 sq. ft.

Page 96. — **5.** 314.16 sq. in. **6.** 254.4696 sq. in. **7.** 153.9384 sq. in. **8.** 95.0334 sq. in. **9.** 314.16 sq. in.

Page 97. — **11.** 12.5664 in. **12.** 5 in.

Page 99. — **6.** 69.28 rd. **7.** 80 min.

Page 100. — **8.** 5.125 sq. ft. **9.** 4 ft. **10.** 24 ft. **11.** 150 rd. long; 100 rd. wide. **12.** 96 rd. **13.** 44 rd.

Page 102. — **2.** 697.43+ cu. ft. **3.** 137.04+ cu. ft. **4.** $18\frac{1}{2}$ cu. ft. **5.** 112 sq. ft. **Lesson 63.** — **1.** 18.8496 ft.; 113.0976 sq. ft.; 113.0976 cu. ft.

Page 103. — **2.** 268.0832 cu. ft. **3.** 30.4896 cu. in.

Page 105. — **10.** 2 in. **11.** $6\frac{2}{3}$ ft. **12.** 6.34+ ft. **13.** 12 ft. **14.** $1\frac{2}{25}$ T.

Page 106. — **1.** 216 sq. in. **2.** 136 sq. ft. **3.** 848.23+ cu. in. **4.** 100 ft. **5.** 30.59+ ft. **6.** 140 ft. **7.** 16 ft. **8.** 55 A. 88 sq. rd. 26 sq. yd. 8 sq. ft. **9.** 10 ft. **10.** 14.14+ in. **11.** 672.09 gal.

Page 107. — **13.** 11.31+ in. **14.** $157.08. **15.** 18 in. **16.** 32.56+ gal. **17.** $\frac{7}{12}$ hr. **Lesson 66.** — **1.** 80 rd. long; 60 rd. wide. **2.** 16 in. **3.** 209.21+ cu. ft. **4.** 237.37+ sq. ft.

Page 108. — **5.** $106.94. **6.** $3.09. **7.** 38 ft. **8.** 16. **9.** 52.201 ft. **10.** $209.86. **11.** 3. **12.** 10.19+ ft. **13.** 1944 sq. ft. **Lesson 67.** — **1.** $3600. **2.** $325. **3.** 7%.

Page 109. — **4.** $\frac{1}{4}\frac{8}{5}\frac{1}{0}$. **5.** 12¢, loss. **7.** $298\frac{2}{3}$ sq. yd. **8.** 232 ft.; $77\frac{1}{3}$ yd. **9.** 24 ft. **10.** $73\frac{1}{3}$ sq. yd. **11.** $43200; 9600; $42984. **Lesson 68.** — **2.** $\frac{2}{3}$; $\frac{2}{3}$. **3.** .12½ A.

Page 110. — **6.** $719.25. **7.** 22.797+ bbl. **8.** 113.137+ rd. **10.** 523.6 cu. in.; 314.16 sq. in. **11.** $760.91. **Lesson 69.** — **1.** 301.593 sq. ft. **2.** $4\frac{1}{2}$%. **3.** $\frac{1}{1}\frac{2}{6}\frac{3}{7}$.

Page 111. — **6.** $10, loss. **7.** 2. **8.** £575 7*s.* 2.9+*d.* **9.** $180, J.; $150, H. **10.** $12.881. **11.** 14 hr. 41 min.

Page 114. — **1.** 40 dm.; 55 dm. **2.** 350 cm. **3.** 45.6 Dm. **4.** 38.69 dm. **5.** 350 Dm. **6.** 52.43 m. **7.** $1.16. **8.** 340 cm. **9.** $2. **10.** $11.75. **11.** 9087 mm. **12.** 8700 cm. **13.** 14.6324 Km.; 14632.4 m. **14.** 36.849 Dm.; 36849 cm. **15.** 8408.7 m. **16.** 91.554 m. **17.** 6.86 m. **18.** 22.2 m. **19.** $174.40. **20.** $2.

Page 116. — **1.** 386.49 Ha. **2.** 8964500 sq. cm. **3.** 2214 sq. m.; 22140000 sq. cm. **4.** 89634000000 sq. mm.; 896.34 sq. Dm. **5.** $798. **6.** 536.84 Ha.; .38645 A. **7.** 896.45 sq. Dm. **8.** 419.24 ca. **9.** $3400. **10.** 22.7 m. **11.** 51129 sq. yd. **12.** 9.03 A. **13.** 37 A. 77.1714 sq. rd.

Page 117. — **14.** 3642.036 A. **15.** 89.04 m.

Page 118. — **1.** 8500 cu. mm.; 4500000 cu. mm. **2.** 95.6 cu. m. **3.** 421372 cu. dm.; 75.006 st. **4.** .13684 cu. Dm.; 136840 cu. dm. **5.** 7.85 cu. m. **6.** $27.30. **7.** $55.31¼.

Page 119. — **8.** $8.64. **Lesson 73.** — **1.** 73.6 dl.; 7360 ml. **2.** 97.02 l. **3.** 53.86 l.; .5386 Hl. **4.** 72346.7 l.; 723467 dl.; 72346700 ml.

Page 120. — **5.** 8.364 l. **6.** 2.3328 l.; 8.1 ml. **7.** $8.88. **8.** 9 Hl. 4 Dl. 5 l. **9.** $9.54. **10.** $6.076.

Page 121. — **1.** 2964.31 g.; 29.6431 Hg.; 2.96431 Kg. **2.** .07634 T.; 763.4 Hg.; 76340 g. **3.** 7057.054 g. **4.** 36.288+ g. **5.** 2268.01+ g. **6.** 1181.6656 lb. **7.** 115.76 g. **8.** 23.855 Hg. **9.** $2.064. **10.** 500. **11.** 12.7 T. **12.** 88.9 Kg.

Page 122. — **1.** 196850 in. **2.** 50.391 m. **3.** 36.67 m. **4.** 365.6 m. **5.** .392 m.

Page 123. — **6.** 46.224 Km. **7.** 7264.38 A.; 72.6438 Ha. **8.** 80000 ca. **9.** $73.60. **10.** 5906.25 Kg. **11.** 2099.5 Hl. **12.** 41.6052 m.; 5.6325 Km. **13.** $7.078. **14.** 25l.

Page 124. — **2.** $1\frac{5}{12}$ hr. **3.** $2\frac{2}{3}$ da. **4.** $1\frac{17}{48}$ hr. **5.** 30 da. **6.** $2\frac{2}{7}$ da. **7.** 24 da.

Page 125. — **8.** $9\frac{3}{23}$ da. **Lesson 77.** — **2.** $\frac{1}{10}$ mo. **4.** $\frac{3}{13}$ da. **5.** $9\frac{3}{23}$ da. **6.** 36 da., A; 18 da., B; 18 da., C. **7.** $5\frac{1}{2}$ da.

Page 126. — **4.** $21\frac{9}{11}$ past 2. **5.** $32\frac{8}{11}$ min. past 3. **6.** $38\frac{2}{11}$ min. past 1. **8.** $43\frac{7}{11}$ min. past 4. **9.** $10\frac{10}{11}$ min. past 5.

Page 127. — **1.** 140° F. **2.** 161.6° F. **3.** $29\frac{4}{9}$° C. **4.** $22\frac{2}{9}$° C. **5.** $22\frac{2}{9}$ C. **6.** 64.8° F.

Page 128. — **1.** $559\frac{3}{8}$ lb. **2.** 450 lb. **3.** $548\frac{3}{4}$ lb. **4.** $103\frac{3}{4}$ lb. **5.** 7.83. **6.** 11.35. **7.** 198.75 lb.

Page 129. — **2.** 240; 320. **3.** $567, J.; $384, H. **4.** 160 bu., A; 300 bu., B. **5.** 360 boys; 525 girls. **6.** A's = 180 A.; B's = 250 A. **7.** $72, B; $112, A. **8.** B's = 80 A.; C's = 198 A. **9.** $7.75.

Page 130. — **10.** $2800, W.; $4000, J. **11.** B does $\frac{1}{18}$ in 1 da.; C does $\frac{1}{9}$ in 1 da. **12.** 960; 480. **Lesson 82.** — **2.** $600, A; $800, B. **3.** $84; $56. **4.** $100, A; $160, B. **5.** $3, Wilson; $8, Brown.

Page 131. — **6.** $60, A; $48, B. **7.** $500, M; $800, N. **9.** $12; 6 yd. **10.** $75; 10 sheep.

Page 132. — **2.** 10 yr., son; 40 yr., father. **3.** 5 yr., Thomas; 25 yr., father.

Page 133. — **5.** 24 in. **6.** 20 yr., daughter; 40 yr., mother. **7.** $252, John; $324, James; $424, Henry. **Lesson 84.** — **1.** $5280, first; $2640, second; $880, third. **2.** 30. **3.** 180. **4.** 32 bu., W.; 96 bu., R.; 288 bu., C.

Page 134. — **5.** 4 nickels; 24 dimes; 36 cents; $2.96. **6.** $438, A; $146, B; $438, C. **7.** 40 yr., father; 10 yr., son. **8.** 40 yd. **9.** 24 bu., A; 4 bu., B. **10.** $2486, A; $1808, B. **11.** 3960; 4752.

Page 135. — **2.** 8. **3.** 12 yd. **5.** 1 ¢, apple; 4 ¢, orange. **6.** $5, calves; $20, cows.

Page 136. — **1.** 12 da. **2.** $146\frac{2}{3}$, A; $133\frac{1}{3}$, B. **3.** $\frac{3}{13}$ da. **4.** 105 mi. **5.** 86° F. **6.** $32\frac{2}{9}$° C. **7.** $21\frac{9}{11}$ min. past 1 o'clock. **8.** $38\frac{2}{11}$ min. past

4 o'clock. **9.** $5\frac{5}{11}$ min. past 7 o'clock. **10.** 8 hr. 2 min. 41 sec. **11.** 62° 33′ 45″

Page 137. — **12.** 84° 26′ west. **13.** 901; 4110; $\frac{6}{25}$. **14.** 360; 108.

Lesson 87. — **1.** 8.373+. **2.** 8.4. **3.** 2.65. **4.** 8 yd. **5.** 35937 sq. ft. **6.** Rectangle is 10 rd. longer. **7.** 39.734+ sq. yd.

Page 138. — **2.** $5\frac{7}{10}$ mo. **3.** $5\frac{1}{7}$ mo. **4.** $46\frac{2}{3}$ da. or 47 da.

Page 139. — **6.** $3\frac{1}{3}$ mo. **8.** $1\frac{1}{2}$ mo. **10.** $10\frac{1}{2}$ mo.

Page 140. — **2.** April 11. **3.** Aug. 28. **4.** Oct. 18. **5.** $3\frac{3}{5}$ mo. **6.** 19 mo.

Page 142. — **2.** $3678.60. **3.** $4227.868. **4.** $773.824. **5.** $920.927. **6.** $2585.869. **7.** Compound interest is $4.645 more.

Page 144. — **2.** $1830.56. **3.** $2291.157. **5.** $2323.585.

Page 145. — **7.** $871.70. **8.** $7513.38. **9.** $2663.40.

Baird, S.W. 66565
Graded work in arithmetic. 8th year.

NAME	DATE	NAME	DATE